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THE *SECRET* WORLD OF LARGE NUMBERS

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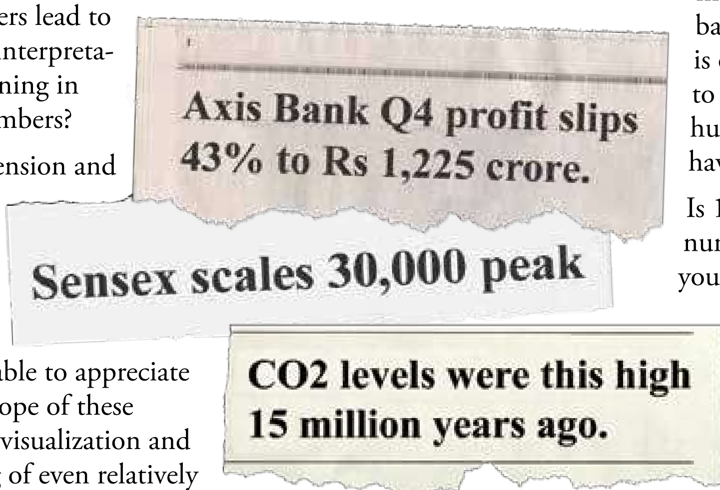
The secret world of large numbers.

Increasingly, we live in a world where large numbers seem to bombard us from all directions. Thanks to globalization and knowledge explosion, encounters with very large numbers happen on a regular basis. Yet, one pertinent question is: 'Does increased exposure necessarily lead to an understanding of the size of these numbers?' Or is it possible that we underestimate the size of these due to their constant use! Does excessive familiarity prevent right perspective? Also, will not a lack of understanding of large numbers lead to an ineffective interpretation and reasoning in using such numbers?

Our comprehension and the students' understanding of the world remain incomplete unless we are able to appreciate the size and scope of these numbers. Yet, visualization and understanding of even relatively small numbers like ten thousand and a lakh (hundred thousand) may be difficult for many people. When does one see ten thousand units of some object? Do one's every day experiences include such numbers?


Axis Bank Q4 profit slips 43% to Rs 1,225 crore.

Sensex scales 30,000 peak

CO2 levels were this high 15 million years ago.

Do such numbers arise in the child's experience of life? If not, how does the teacher help the child to mentally visualize such numbers? Large numbers are incomprehensible to students unless they are able to relate these figures to their own experiences, say of crowds, etc. We often hear statements like 'there were thousands of people at the rally' when the numbers may well have been a few hundreds. Is that a lack of skill of estimation? Is it a lack of sense of a thousand?

Now we will get to the question: When does a number get viewed as a large number? That is relative and it is context dependent. Is 10 a large number? It doesn't seem so. However we are aware that the brain can recognize up to 4 or 5 objects without counting. It would not be able to recognize 10 objects at a glance. 10 is too big for the brain to do that!

However, all of us recognize that 10 is a very important number for us as our decimal system is based on 10. This is obviously due to the fact that human beings have 10 fingers.

Is 100 a large number? For many young students 100 seems quite large. And they are proud of being able to rattle off the numbers up to 100. A hide and seek game is often started off by a count of 100. 100 runs is an achievement if one is a batsman. 100 certainly is impressive! 100 does give us a sense of largeness and yet it is humanly attainable. One can skip 100 times, run a 100 metre stretch easily. We routinely talk about distances in hundreds of kilometers. We purchase clothes for hundreds of rupees.

How about a 1000? Nobody would embark on counting up to a 1000. It is too large to do that. 1000 is also a huge deal in our world and is called by different names: a kilo, and k or G. A 1000 rupee note (in the days before demonetization) was considered too big to be used while paying for a bus

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ticket or an auto ride. However many things are priced in thousands. It is large, yes, but it is also fairly common-place.

Interestingly the Roman numerals go up to a thousand (I V X L C D M).

Is a lakh a large number? Or is a million a large number? Yes, these are very big numbers but they are not beyond human comprehension. We talk about populations in millions, distances in the solar system in millions of kilometers, prices of houses in lakhs and crores.

Yet at some point as students begin to explore the secret world of numbers, they will meet billions, trillions and even bigger, fearsome, awe inspiring numbers which have majestic names and are too difficult to comprehend. They will meet such numbers in physical sciences (astronomical distances, atoms), in life sciences (number of cells, number of blood cells, RBC or WBC in a drop of blood). There is a need for benchmarks to comprehend the magnitudes of such large numbers.

So, how do we help students in developing a sense of large numbers?

While introducing large numbers to students what are our aims?

• Skill of reading and writing large numbers, of course. However, merely helping students with procedural skills of reading and writing large numbers does not aid in building a number sense. They need to appreciate the size of a number and an understanding of the relative sizes of numbers.

• Understanding the relationships between different places. Our number system is a language describing quantities and relationships between quantities. Students need to be able to understand and use these relationships flexibly and with ease. For example: They need to understand that 10 thousands = 100 hundreds = 1000 tens = 10,000 ones. A million is 1000 thousands. It is a thousand times a thousand.

• To make sense of large numbers in context, whether it is in science, social science or business.

• To meet students' natural fascination for large numbers.

• More importantly to develop their mathematical and logical power in building an appreciation of such numbers.

Here is a set of questions we can work with to enhance procedural skills, relational skills and a sense of the size.

How does our knowledge of place value relate to the way we read large numbers? How is the value of a digit determined in a place value system? How are the different places in a place value system related to one another? How much bigger is a crore than a lakh? How much smaller is a thousand than a million? In what ways does estimation assist in appreciation of the size of a number? In what way does the usage of a bench-mark, a reference point, useful in understanding the magnitude of a number?

Introduction of Numbers at the Upper Primary Level.

Revisit quickly the usage of standard materials (cubes, flats, longs, units) to represent numbers up to a thousand (by drawing them on the board). It is important not to assume that students will remember the conceptual basis of the place value system. Remind them that the value attributed to a digit depends on its place with reference to the rightmost place (units). Review and establish the principle of relationship, that each place is ten times the place preceding it.

Once students have understood the grouping pattern till ten thousand, they will not have difficulty in extending the place value system. In the Indian system, the naming is in blocks of two (thousand, ten thousand and lakh, ten lakh, etc.) Are the students learning to apply the rules of the place value system (each place is ten times the place to its right) to understand the value of the new places?

In the international system, the naming of these places follows a simple pattern in blocks of three (thousand, ten thousand, hundred thousand and million, ten million, hundred million, etc.) However, note that the term billion has two different meanings. A billion is 1000 million in the American system. The European billion is a million million.

Students will naturally become curious and ask 'What is the next place called? How far can we go?'

At this point they can use the internet to find the names of some interesting numbers. What number is one followed by a hundred zeroes?

Discussion can also lead to 'What is the largest number?' The discovery that numbers will never end can come as a great surprise! Let students come upon it on their own.

Sometimes students hold a mistaken notion that infinity is the largest number. Help them to understand and distinguish between the concept of infinity and very large numbers! Infinity is not a number. It means that numbers continue endlessly. Link it to other contexts of infinity. Example: How many points can be found between any two given points?

As students begin to explore astronomical distances, they will encounter a new measure: the light year. Discuss the problem of expressing the huge distances between stars and galaxies and the historical need to invent light years. A light year is the distance light travels in a year; it is slightly less than 10 trillion kilometers.

Exposure to exponential numbers, number representation in calculators and scientific notation will be the next step on the ladder.

TEACHING AIDS

Aid A: Reading in stages!

Objective: To build the skill of reading number names.

Materials: Large number cards as shown in the picture (The example here is in Indian system.)

Note: Thousands and ten thousands together are considered as a family of thousands, similarly lakhs and ten lakhs together belong to the family of lakhs, etc. They are read together.

Students are already familiar with reading numbers up to ten thousand or a lakh by this stage. While introducing lakh and crore, one can start by helping students to read numbers with which they are already familiar, and then name the new places (lakh and crore).

Open the card in stages as shown, revealing hundreds, tens and units first. (017) Read seventeen.

Follow it up by opening thousands block (18,017) (note: both thousands and ten thousands are read together). Read it as eighteen thousand and seventeen.

Follow it up further by opening up the lakhs block (both lakhs and ten lakhs are read together).

(02,18, 017) Read it as two lakh, eighteen thousand and seventeen.

Finally open the full number (3,02,18,017). Read it as three crore, two lakh, eighteen thousand and seventeen.

Aid B: Reading in blocks (Example in international system.)

Objective: To facilitate reading in blocks.

This is according to the American system where one billion equals 1000 million.

Materials: Place value charts as shown in the picture.

Use structured coloured reading charts to help in learning, to read in blocks (in stages).

Note: In the international system thousands, ten thousands and hundred thousands belong to the family of thousands. Similarly millions, ten millions, hundred millions belong to the family of millions, etc.

6,247,148.

Help the child to read the digit in the millions place first (from the millions family). In order to identify the place value of 6, students will need to name the places in order from the right side. Read it as six millions.

Thousands family members are read together. Two hundred and forty seven thousand. Read it as six millions, two hundred and forty seven thousand.

Finally the full number is read. Read it as six million, two hundred and forty seven thousand, one hundred and forty eight.

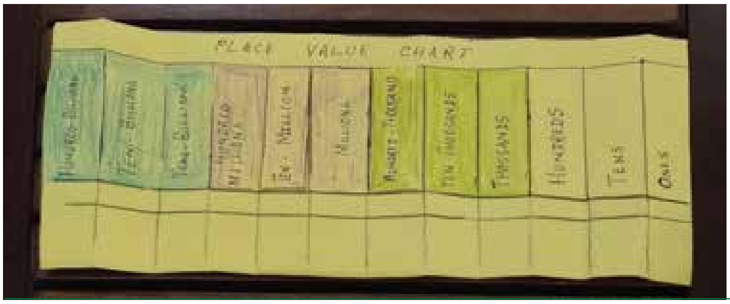
Aid C: Writing in blocks!

Objective: To facilitate writing in blocks.

Materials: Place value charts as shown in the picture.

They will place zeroes in the missing spaces.

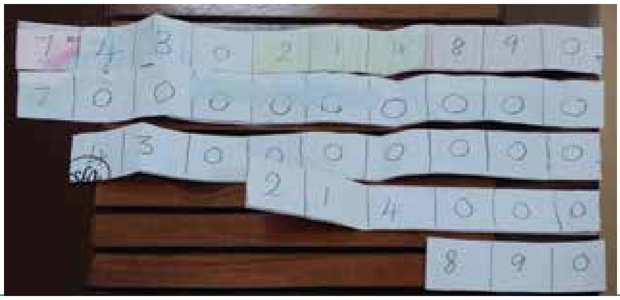
Students should initially be encouraged to use place value charts to record the numbers read out.



Aid D: Decompose!

Materials: Expanded notation cards as shown in the picture.

Objective: To break up the number into its components.



Numbers can be initially decomposed using family groups together.

Later they can be decomposed separately, i.e., hundred thousand separately, ten thousand separately, thousand separately, etc.

Aid E: Build and read!

Objective: Scaffolding for learning to read large numbers

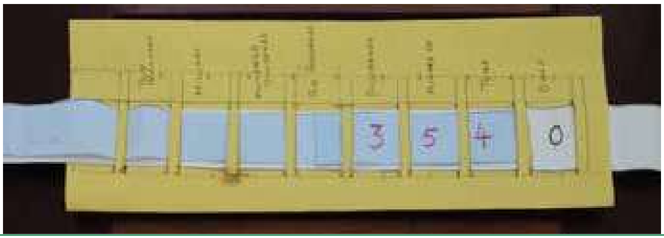
Materials: 3 digit number groups

Students can build numbers and practise reading them. Each set of three digits will be treated as a family while reading.

Aid F: Slide fun!

Objective: To understand the relationships between different places.

Materials: Slide, one background card with zeroes written on it, changeable large number cards in the front as shown in the picture.



Use a slide as shown in the picture. Slide the front card to demonstrate multiplication by 10 or 100 or 1000.

Note: Are all numbers read with place values? Think of phone numbers, registration numbers and identification numbers. They look like very big numbers, but they are actually code words rather than numbers, and place value has no meaning for such objects.

Problem Solving

Here is a set of problems which can help build quantitative literacy and increased number sense. The problems are posed. They require students to do mathematics! (This means: make some assumptions, discuss and use resources, if necessary, and then compute). The students see that the results apply to real world situations and are obtained by estimation and approximations. They can also use the estimations to check the reasonableness of a given answer. While solving these problems, students should develop their own steps and methods. The process of organized thinking in developing strategies to reach an estimate becomes more important than the actual answer itself.

'Learning to think algorithmically builds mathematical literacy.' (NCTM)

I have suggested a possible approach which students could follow only for activity 1 and 3. It is of the utmost importance to let the students work out the strategy. It is also equally important to record the assumptions made, the strategy adopted, the approach (the in-between steps) and findings.

ACTIVITY 1: **HOW MANY GRAINS OF RICE DO I EAT EVERY DAY?**

Objective: To build an understanding of the relative magnitude of numbers to a thousand.

Materials: A cupful of rice grains, paper plates

Place 1 grain on the first plate.

Count out 10 grains and place on the second plate.

Pick up approximately 10 grains at a time. (Not counting at this point). Do this ten times to place approximately 100 grains on the third plate.

Now pick up approximately 100 grains at a time. (Not counting). Do this ten times to place approximately 1000 grains on the fourth plate.

At the end the teacher could share the normal measure of rice a person eats per day. Students can pour their 1000 grains into a measuring cup to judge how much they eat in a day.

ACTIVITY 2: **CAN WE GO ROUND THE WORLD?**

Fact: The diameter of the earth is 12,756 kilometers.

Objective: To build an understanding of ten thousand.

Problem: Consider ten thousand students holding hands and standing in a line.

How far would they reach?

Would they go across a football field?

Across your state?

Across India?

Around the world at the equator?

ACTIVITY 3: **HOW MANY ARE WATCHING CRICKET?**

Fact: Before its renovation in 2011, India's Salt Lake Stadium seated 1,20,000.

Objective: To build an understanding of a lakh.

Approach: Students can count the number of people their classroom can seat or estimate the number of people the school assembly hall can seat. They could find out the capacity of a theatre or auditorium they are familiar with and proceed from there. They can use rounding where necessary.

Discuss students' suggestions for finding out the answers.

Encourage them to record their findings in writing.

1,20,000 is roughly the same as 2,000 of our classrooms.

1,20,000 is roughly equal to the number of seats in 600 theatres.

ACTIVITY 4: **MESSY HAIR!**

Fact: People have about 100,000 hairs growing on their heads!

Objective: To build an understanding of a hundred thousand.

Materials: A few graph papers from a millimeter graph book

Give students a millimeter graph paper.

Problem: Find out how many sheets would be needed to have 100,000 little (millimeter) squares.

ACTIVITY 5: **REACHING THE MOON!**

Fact: Our moon is at a distance of 3,84,400 km from the earth.

Objective: To build an understanding of a lakh in the context of height.

Problem: Consider 1 lakh students, each standing on the shoulders of another.

How high would they reach?

Would they be as high as the Qutub Minar?

As high as the tallest building in the world?

As high as a satellite?

As high as the moon?

ACTIVITY 6: **HOW MUCH SPACE?**

Objective: To build an understanding of a million in the context of capacity.

Materials needed: A set of cubes

Problem: Estimate how much space a million cubes would occupy.

Would they fit in the cupboard?

Would they fit in the class room?

Would they fit in the assembly hall?

ACTIVITY 7: **HOW HEAVY?**

Fact: The blue whale weighs approximately 140,000 kilograms.

An Asian elephant weighs 5,500 kilograms.

Objective: To build an understanding of a hundred thousand in the context of weight.

Can students use knowledge of their own weight to understand these stupendous weights?

ACTIVITY 8: **AM I WELL-READ?**

Objective: To build an understanding of a hundred million.

Materials: Printed paper from any book.

Give students a paper printed fully with text.

Problem: Find out how many books would be needed to have read a hundred million letters (characters).

Once the activity is over, they can assess how well-read they are!

ACTIVITY 9: **HOW OLD AM I?**

Objective: To build an understanding of a million and billion in the context of time.

Problem: How old are you if you have lived for a million seconds?

How many seconds old will you be on your 10th birthday?

Does anyone live for one billion seconds?

ACTIVITY 10: **IF I WERE RICH!**

Objective: To practise writing big numbers while writing cheques.

Problem: How will your class spend a million rupees?

A problem like this can be placed in a context. Your class is going on an expedition to Antarctica for one week. What do you need for the expedition? Food? Clothing? Gadgets?

Students can get data about prices of different things from a wish list.

ACTIVITY 11: **WALKING IS THE BEST!**

Objective: To develop a sense of million in the context of distance.

Problem: Will I ever walk a total of a million km?

How will students find out their walking rate? How many hours can a person walk per day? Are the students realistic in their problem solving approach?

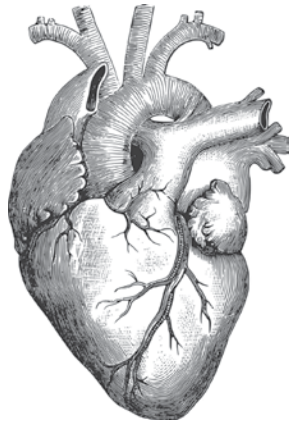
ACTIVITY 12: **MY HEART IS BEATING!**

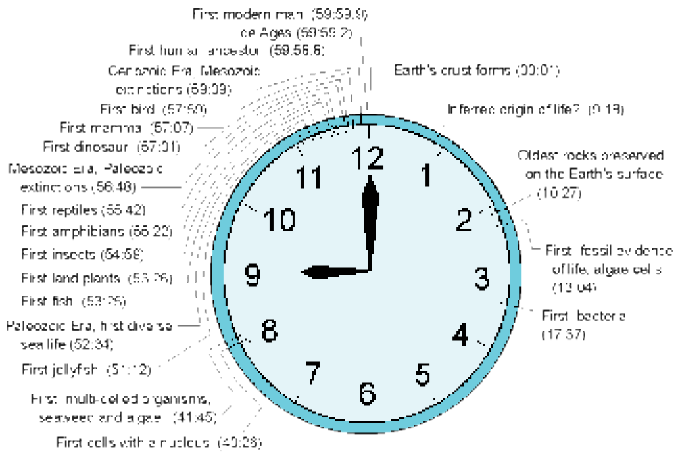
Objective: To develop a sense of billion in the context of time.

Problem: How many times would my heart beat if I live up to 80 years?

What information is needed? How will students use scaling as an approach?

Can the information be obtained by counting beats per minute? Is there an exact answer? Does it vary?





4.6 billion years in one hour

ACTIVITY 13: **HUMAN HISTORY TICKING AWAY!**

Objective: To develop the skill of using a 24 hour clock to represent the arrival of human beings on earth. (There is an implicit usage of the notions of scaling and proportion here).

Problem: Students can construct a 24 hour clock to depict history of earth and the recent arrival of man on earth.

What information is needed? How many years are represented by each hour?

When did the earth have dinosaurs? In terms of hours, how long ago is that?

What is the relationship between a place value system and the number of digits it uses?

The Hindu- Arabic numeration system uses base 10, notion of place value and zero in a dual role, as a digit as well as a place holder. The Hindu-Arabic numeration system allows very large numbers to be written.

Point out the way the Roman number system works. In the Roman system a few letters represent certain quantities. Each letter stands for the same value no matter where it is placed. Zero is not represented in any form.

I II III IV
V VI VI VIII
IX X XI XII
L C D M

ACTIVITY 14: **CONTRAST ROMAN NUMBERS AND HINDU ARABIC NUMBERS!**

Objective: To appreciate the efficiency of the Hindu-Arabic system.

Pre-requisite: Familiarity with Roman numbers

I = 1, V = 5, L = 50, C = 100 and M = 1000.

If a heavy bar is placed over the numeral that means it is multiplied by 1000.

For example a V with a bar over it would stand for 5000. An M with a bar would be 1,000,000.

Problem: Students can be given a few numbers (up to a lakh) in words and asked to represent them in both the systems. In the process of writing and reading the numbers they will understand the efficiency as well as simplicity of the Hindu-Arabic system.

How would 50,000 be written in the Roman system?

They can also be given some addition problems to solve in the Roman system first and verify using the decimal system.

Example: 23 + 58

How would we write these as Roman numerals?

23 is XXIII and 58 is LVIII.

If we bring them together they become XXIII LVIII.

Now, we have to arrange these in descending order. It becomes LXXVIIIIII.

We notice that there are six I's. That is the same as VI.

Now the number is rewritten as LXXVVI.

Again, there are two V's. That is an X.

Now the number is rewritten as LXXXI.

We can now use the Hindu-Arabic system to check if our answer is right.

Note: At this point, talk about Aryabhata and his greatest contribution, the concept of zero. While he did not use the symbol which we use now for zero, he did develop the place value system which is based on zero.

PROJECTS

Let students research on the following topics:

Dictionary: How many words in the dictionary?

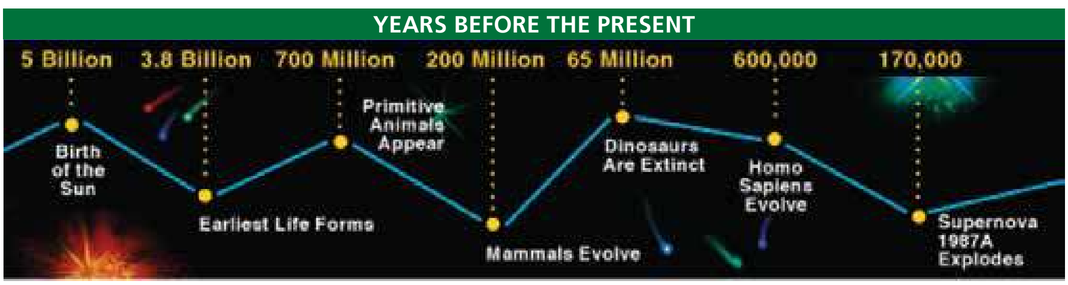
There are about 8,000,000 words in the English language!

Universe: How many stars in a galaxy?

On a perfectly clear night you can see about 2500 stars in the night sky!

Solar system: Building a solar system model (scaled version with all distances and diameters proportional to the actual).

Earth: Age of the earth



Population: Population figures of different states in India.

Human body: Number of cells

One drop of blood can contain about five million red blood cells!

Budget: Allocation of funds to different projects.

Games: Use Monopoly or Business game to give practice in using large numbers in the context of money.

More problems to explore!

How many times is the Qutub Minar taller than you?

How many twelve year olds are there in India?

How many kilometers of railroad tracks are there in India?

How many ants are there in an ant colony?

GAME 1: **Make the number**

Objective: Practice in reading 8 digit numbers

Materials: 3 sets of Number cards with digits 0 to 9, Task cards (about 10)

Tasks can be 'Make the number closest to 5,00,00,000', 'Make the largest 8 digit number', 'Make a 8 digit number which has the largest digit in ten thousands place'.

Number of players: 4

Both number cards and task card are placed face down. The first player selects 8 number cards and one task card from the pile. The player arranges the cards to fit the task best. If another player can suggest a better way of performing the task the first player does not get a point. Else the first player gets a point. Each player takes turns to play. After a few rounds the player with maximum points wins.

Game 2: **Hear! Build!**

Objective: Practice in reading and writing 8 digit numbers in Indian system

Materials: Challenging eight-digit number cards

Number of players: 2

The number cards are placed face down. The first player selects a number card from the pile and reads it out without letting the other player see the number. The second player writes the number down. (Restrictions can be placed: No corrections are allowed after the number is written down). This is followed by the second player selecting a number and the first player writing it down. For every round, a correct entry gets the player one point.

There were 100 zeroes in this pullout; now it's 102; now 103; now 104;…

Reference: Please also refer to: https://sites.google.com/site/largenumbers/home/1-1/new_intro

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