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MATH TALK IN THE CLASSROOM

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Does Math Talk have a place in the teaching of mathematics?

To answer the above question, let us ask ourselves a few more questions. Does a classroom where the teacher asks questions that can be answered with a simple ‘yes’ or ‘no’ provide scope for students to share their true understanding of knowledge? Does it throw light on their misconceptions? Is there an opportunity for students to learn from one another? Does it expose students as to how their peers look at the question? The answer is obviously ‘no’.

Here are a few important ways in which a student can benefit from a classroom engaged in math talk.

Students will:

- Learn to reason mathematically and articulate clearly, justifying their answers
- Develop self-reliance in testing the correctness of their reasoning
- Engage in stimulating peer interactions and collaborative work
- Develop a deeper appreciation of concepts and processes
- Retain their learning due to this more in-depth engagement

Is math talk possible at the primary level? Are these objectives too ambitious for students who are being introduced to mathematics? If the approach to teaching mathematics in the primary classroom is top-down, teacher-centric, and authoritarian, then it is highly unlikely that children can switch to a different approach in middle school. This may mean that they never develop the confidence to reason, make mistakes, check their own work and learn to try different methods.

Right from Class 1 onwards, teachers can encourage students to demonstrate using objects and informal conversation why 2 and 3 make 5, why 7 is more than 4, etc.

They can also pose questions to get students to express their thinking.

In how many ways can this blank be filled? Why do you think so?

$8 > \_$

Can you show using counters that 2 × 4 is the same as 4 × 2?

Why should multiplying a number by 4 give the same answer as multiplying the same number by 2 and then multiplying the answer you got again by 2?

Many teachers may feel diffident about trying such a format in their classes due to a fear of loss of control over the class, or lack of conviction in such a process as a way of learning. Also, there is a lack of clarity on how to proceed. However, it is possible to conduct classes in which meaningful math talk happens. An illustration of such a discussion from the NCERT book, Class 5, is shown in Figure 1. The discussions listed in this article portray such scenarios.

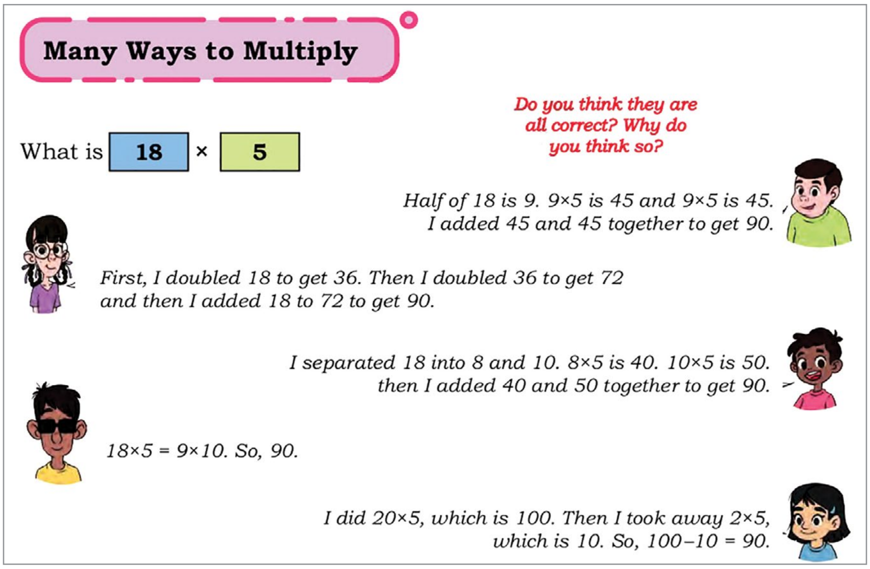


Figure 1: Reproduced with permission from NCERT Class 5 textbook.

What can a teacher do to create such spaces and support productive math talk?

In this article, we will explore a few discussions centred on questions that lend themselves to math talk among peers. These discussions can be organised in different ways.

Figure 2

Source: AI Generated

The teacher can get pairs of students to work together on a problem. They may solve it independently but check with each other if they got the same answer. Or the teacher can get students to work in groups of four sitting around a table, using hands-on material when required. The teacher's prompts will help the students to develop the ability to work together by dividing up the work among themselves.

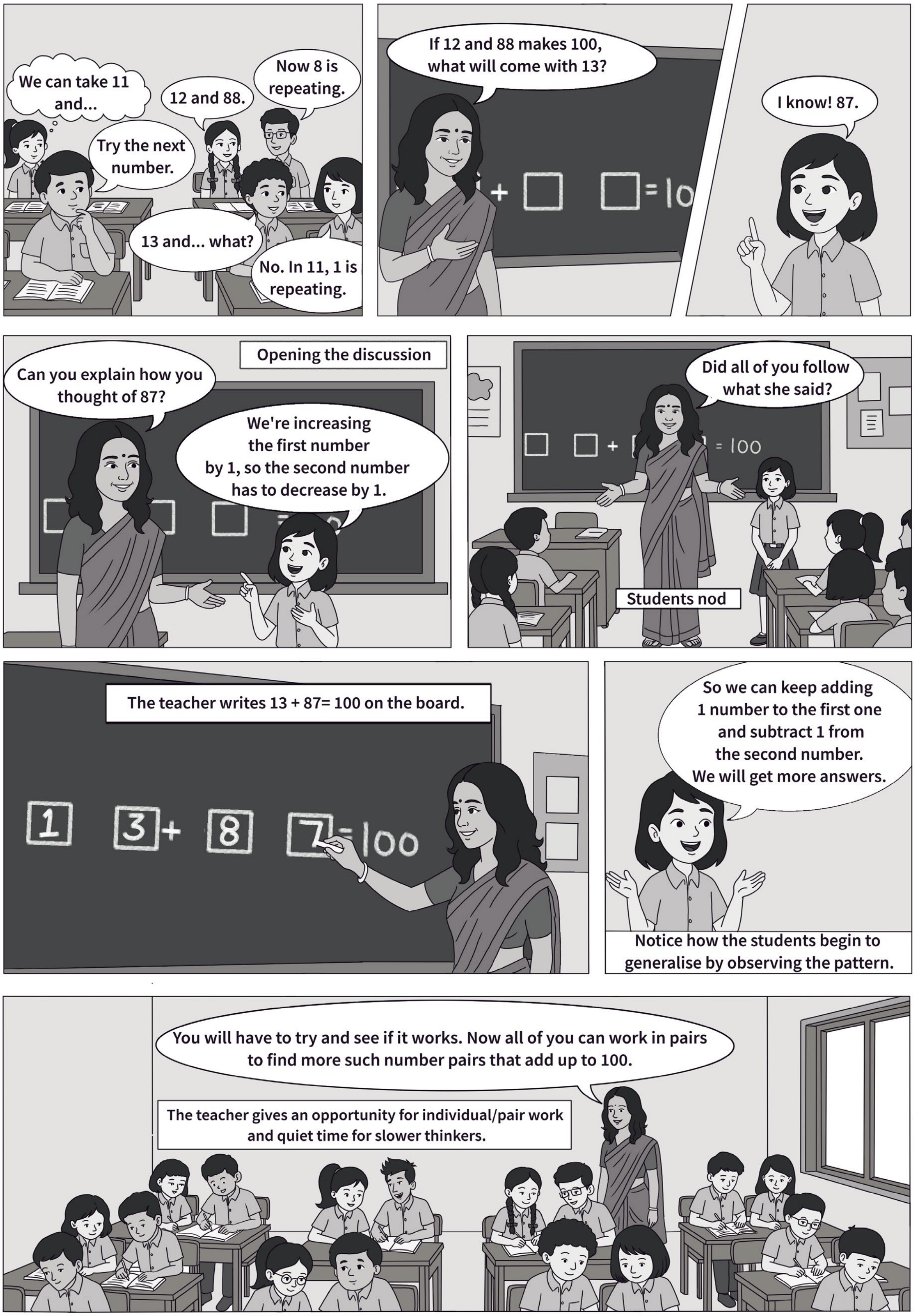
It is good to establish some norms for such a discussion, such as asking them to speak in turns, to listen fully before talking, and so on. This will ensure equal opportunity for all the students to share their thinking and engage in careful listening.

The teacher should pose questions such as *'Can you repeat what he said?'* or *'What did she mean by that?'.* This will develop among the students the skills of rephrasing another student's reasoning and justifying their answers.

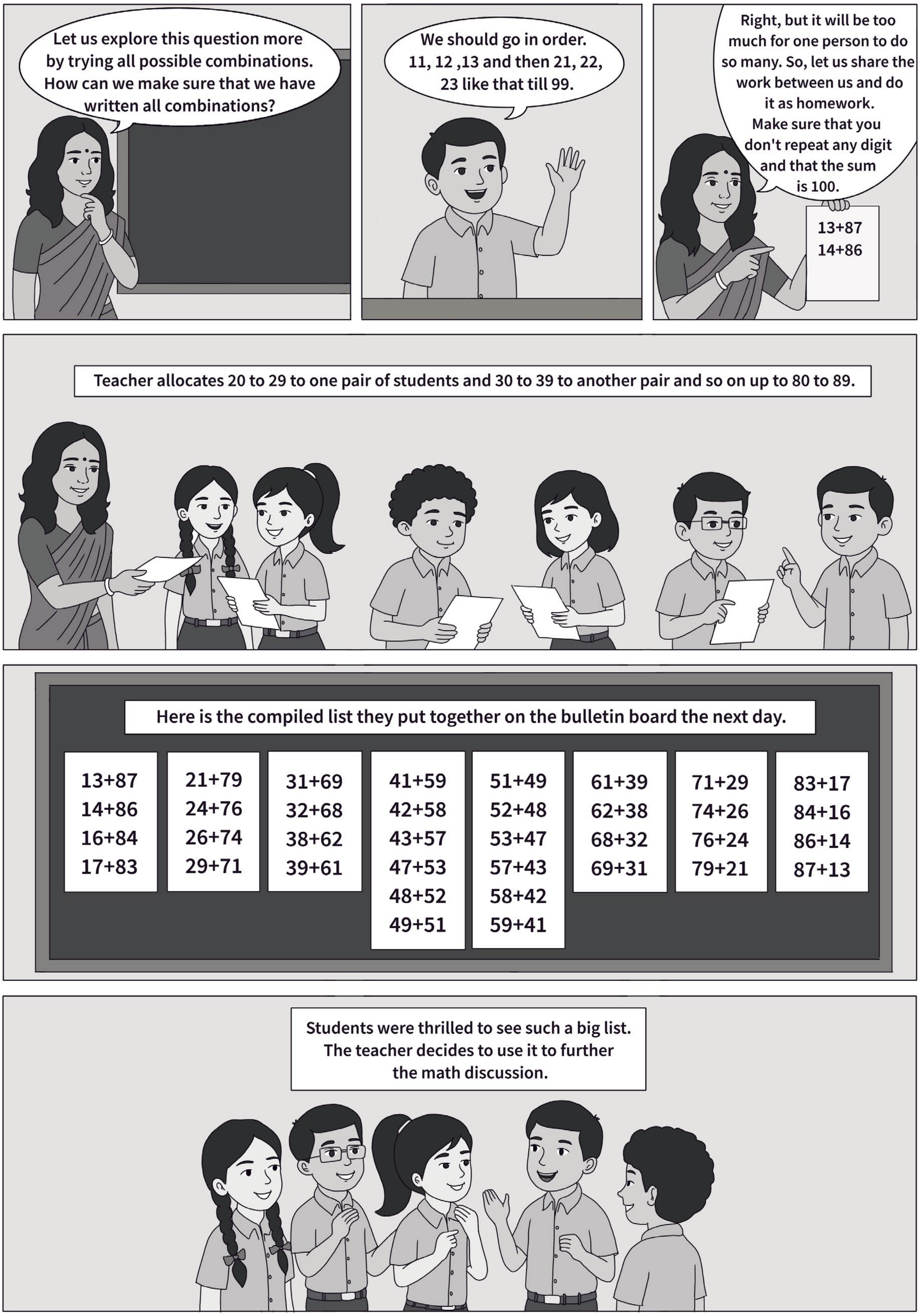
The results of these discussions may be presented to the rest of the class in different ways. The children's written work and drawings can be put up on the bulletin board. If they have come up with a different way to solve a problem, that can be highlighted. Solutions to open-ended problems can be put up. This will help in improving children's appreciation of multiple approaches and make way for peer learning.

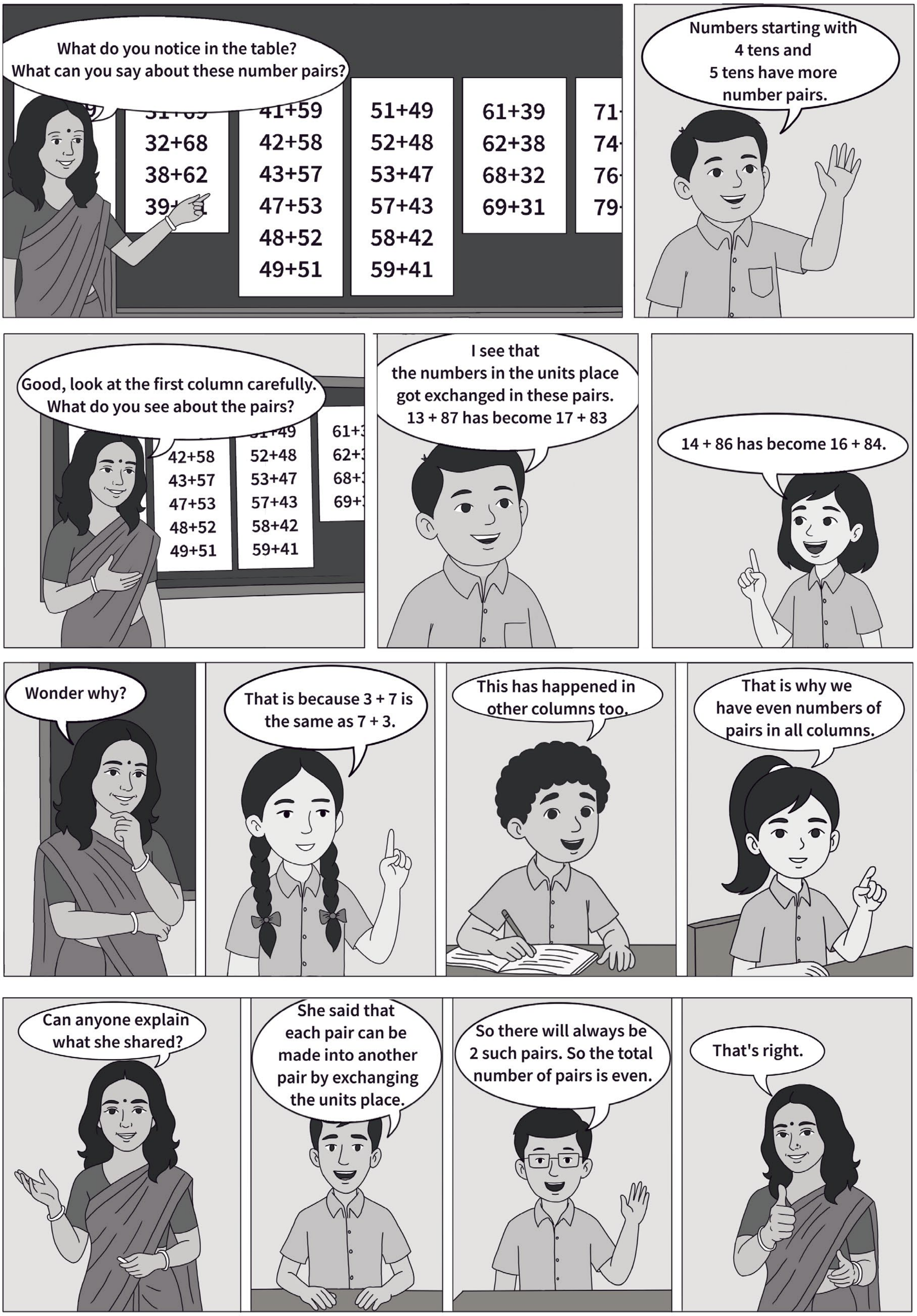
DISCUSSION 1

Grade Level 4, students work in groups of 4.


We can take 11 and...
Try the next number.
12 and 88.
Now 8 is repeating.
13 and... what?
No. In 11, 1 is repeating.
If 12 and 88 makes 100, what will come with 13?
+ □ □=10
I know! 87.
Opening the discussion
Can you explain how you thought of 87?
We're increasing the first number by 1, so the second number has to decrease by 1.
Did all of you follow what she said?
□ □ + □ □ = 100
Students nod
The teacher writes 13 + 87= 100 on the board.
1 3+ 8 7=100
So we can keep adding 1 number to the first one and subtract 1 from the second number. We will get more answers.
Notice how the students begin to generalise by observing the pattern.
You will have to try and see if it works. Now all of you can work in pairs to find more such number pairs that add up to 100.
The teacher gives an opportunity for individual/pair work and quiet time for slower thinkers.

Let's try 14+86, that works.
After that 15+85, but 5 is repeating. So that doesn't work.
But we can try 16+84. That works.
Let's write other numbers also. 17+83. Then 18+82 (No, No).
We can't do 19 and 91 also. That's it. We got all the answers.
Are you sure that you tried all the combinations?
Students are unsure but stay quiet.
Can 2-digit numbers begin with other digits too?
Yes, yes. We can try numbers starting with 21, 22 and so on.
Have you all found some answers? Will you come and write on the board the combinations that you found?
Students write some combinations on the board. Some combinations turn out to be the same. The teacher highlights the unique combinations.


Let us explore this question more by trying all possible combinations. How can we make sure that we have written all combinations?
We should go in order. 11, 12 ,13 and then 21, 22, 23 like that till 99.
Right, but it will be too much for one person to do so many. So, let us share the work between us and do it as homework. Make sure that you don't repeat any digit and that the sum is 100.
13+87 14+86
Teacher allocates 20 to 29 to one pair of students and 30 to 39 to another pair and so on up to 80 to 89.
Here is the compiled list they put together on the bulletin board the next day.
13+87 14+86 16+84 17+83
21+79 24+76 26+74 29+71
31+69 32+68 38+62 39+61
41+59 42+58 43+57 47+53 48+52 49+51
51+49 52+48 53+47 57+43 58+42 59+41
61+39 62+38 68+32 69+31
71+29 74+26 76+24 79+21
83+17 84+16 86+14 87+13
Students were thrilled to see such a big list. The teacher decides to use it to further the math discussion.


What do you notice in the table?
What can you say about these number pairs?
32+68
38+62
41+59
42+58
43+57
47+53
48+52
49+51
51+49
52+48
53+47
57+43
58+42
59+41
61+39
62+38
68+32
69+31
Numbers starting with 4 tens and 5 tens have more number pairs.
Good, look at the first column carefully. What do you see about the pairs?
I see that the numbers in the units place got exchanged in these pairs. 13 + 87 has become 17 + 83
14 + 86 has become 16 + 84.
Wonder why?
That is because 3 + 7 is the same as 7 + 3.
This has happened in other columns too.
That is why we have even numbers of pairs in all columns.
Can anyone explain what she shared?
She said that each pair can be made into another pair by exchanging the units place.
So there will always be 2 such pairs. So the total number of pairs is even.
That's right.


Is there anything else that you notice?
Students are quiet.
What if we look at the fourth and fifth lists together?
Oh! They are backward in the fifth list.
What is backward? What do you mean by backward?
What she means is that the number pairs have come in the opposite way.
Can you give me an example?
In the fourth list, 49 + 51 is the last pair. In the fifth list, 51 + 49 is the first pair.
The number pair is same, but the left and right numbers got exchanged. Does that make a difference to the sum?
No. 3 + 5 is the same as 5 + 3.
Oh! After finding the answers up to 50, we could have just written them the other way round.
It is like symmetry!
That was a good observation that you made.

Further questions can be posed

Do you notice if any digit never appears as the unit digit?

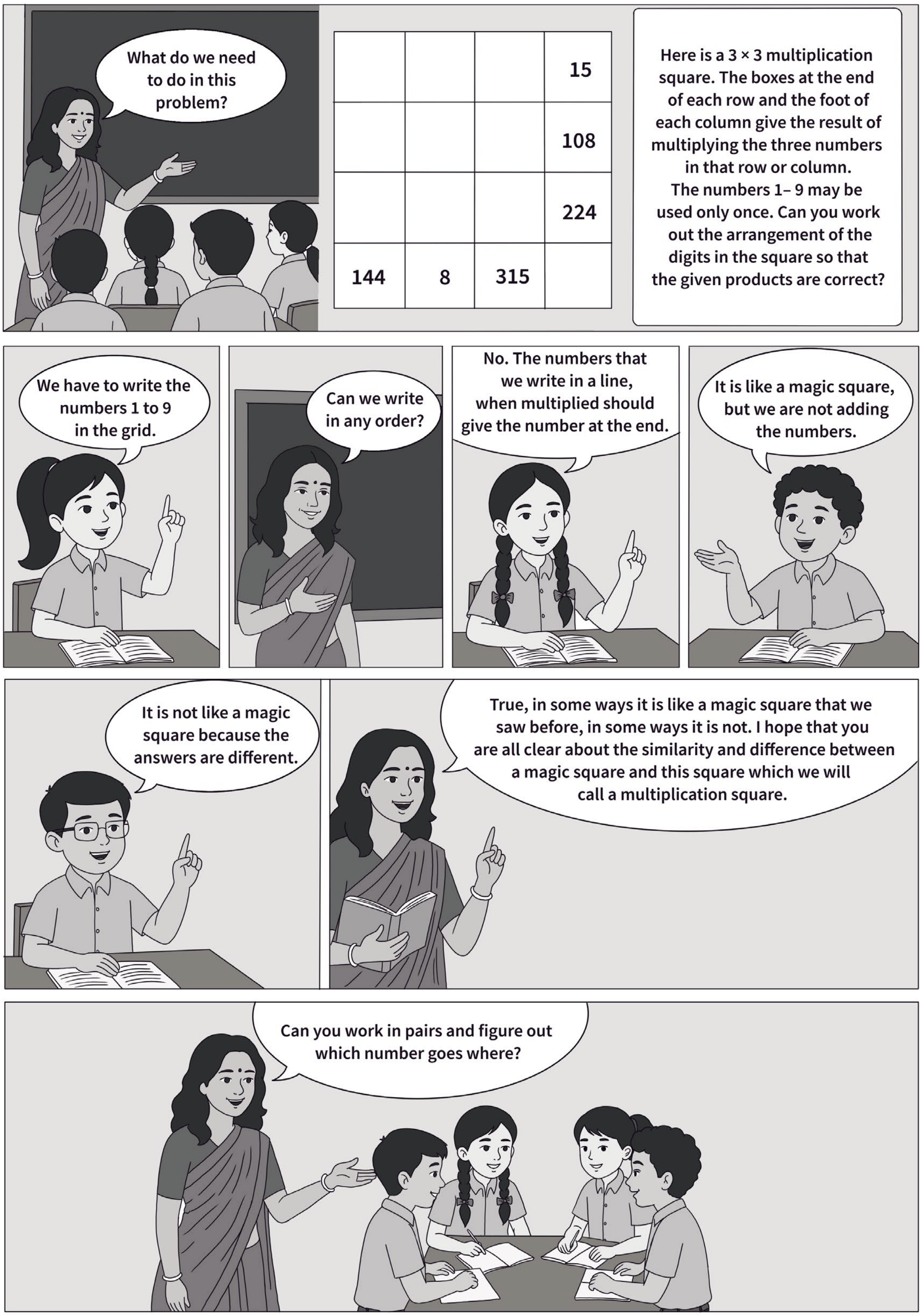
Can you find other ways to find all possible pairs?

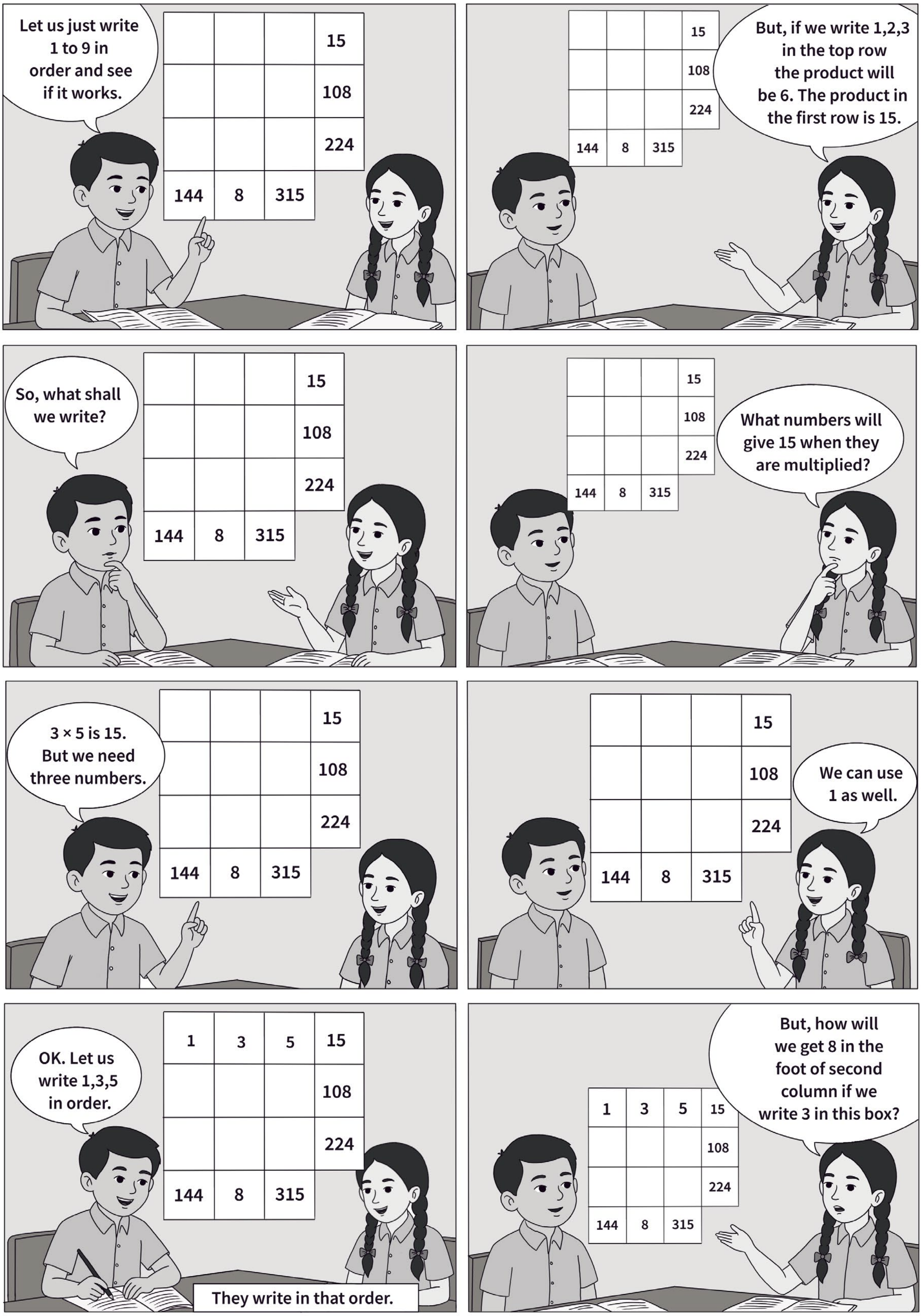
Strategies used by the teacher

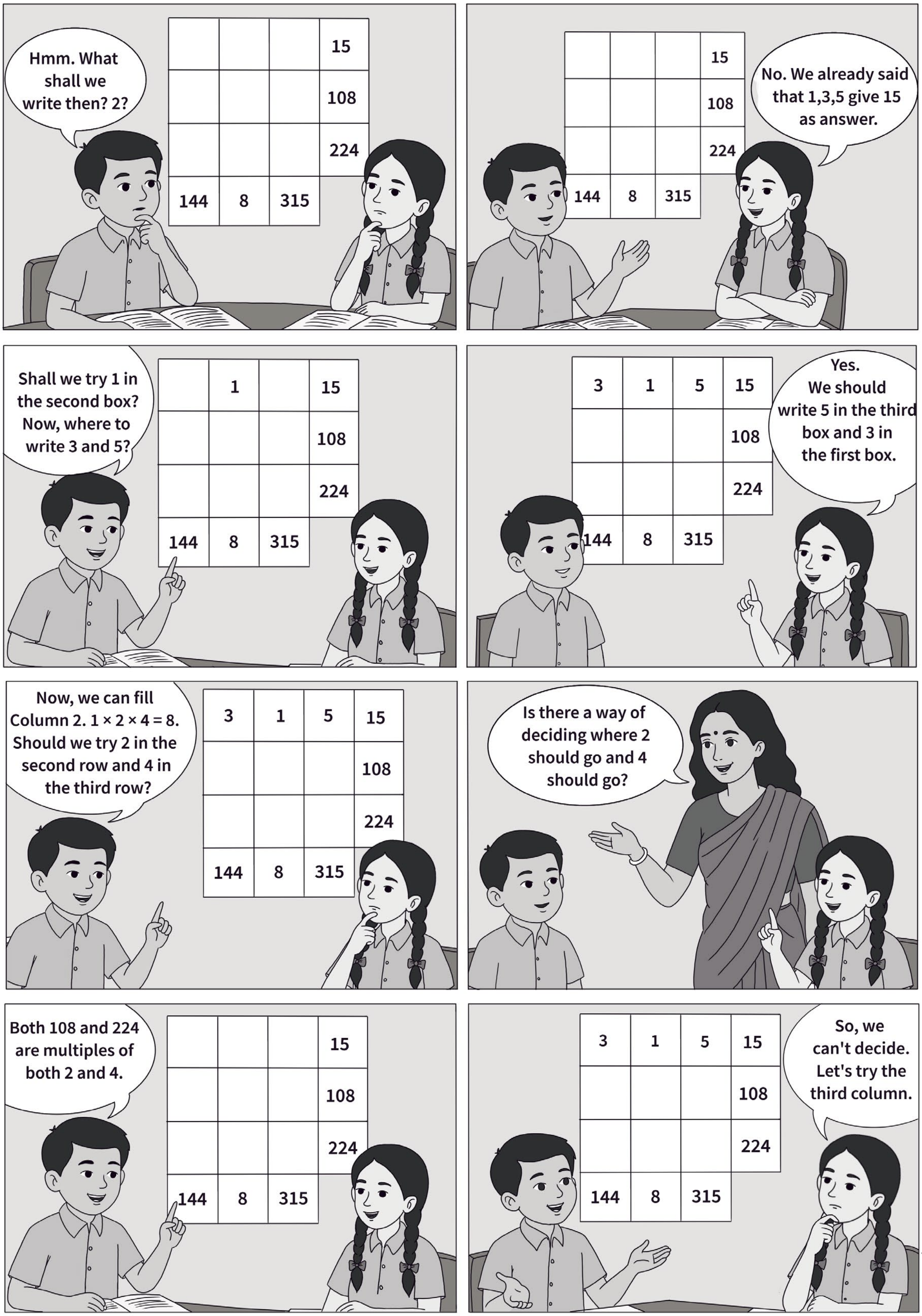
- The teacher used the problem to get students to engage in math talk.
- Students explained the problem in their own words.
- The discussion highlighted the constraints of the problem.
- Together, they reasoned out the answers they got and were provided the opportunity to clarify their understanding of such reasoning.
- Pair work ensured greater participation, particularly among the quieter students.
- The teacher guided the discussion towards a systematic way of finding all possible solutions.
- Homework was an extension of the class work but became an input for individual work and further exploration.
- The students were goaded to search for patterns and find explanations for these. By asking one student to rephrase another's explanation, the class was encouraged to listen to one another. Contributions were valued.
- Usage of appropriate math language was facilitated.
- Arithmetic laws were reinforced and revisited.
- Just one such session could achieve multiple objectives efficiently.
- Did you notice any other strategies that the teacher used?
- In the following discussions, try and pick up some other strategies. What is the teacher doing differently? What is the same? Can you recognise her objectives?

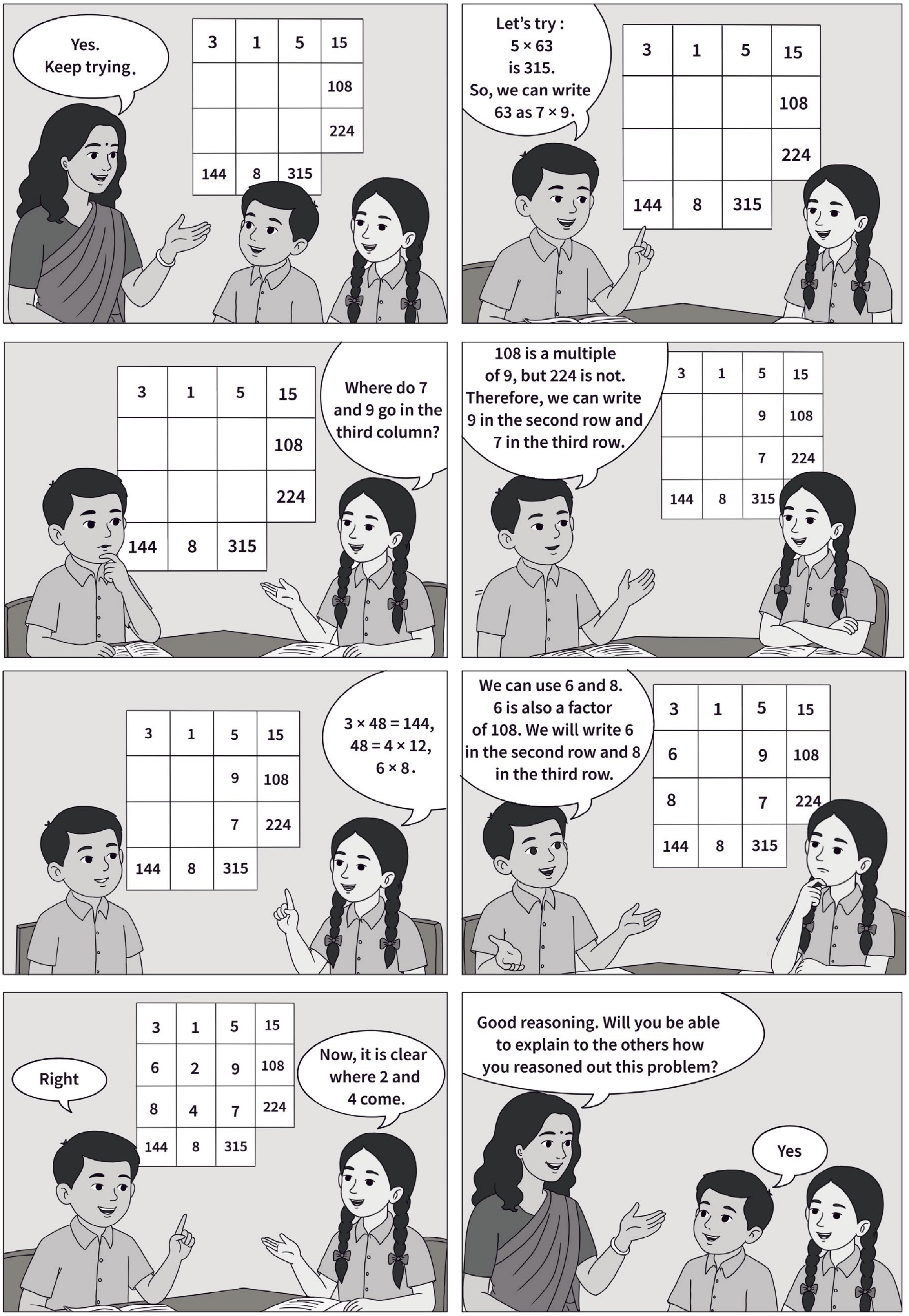
DISCUSSION 2

Grade Level 5, students work in pairs.

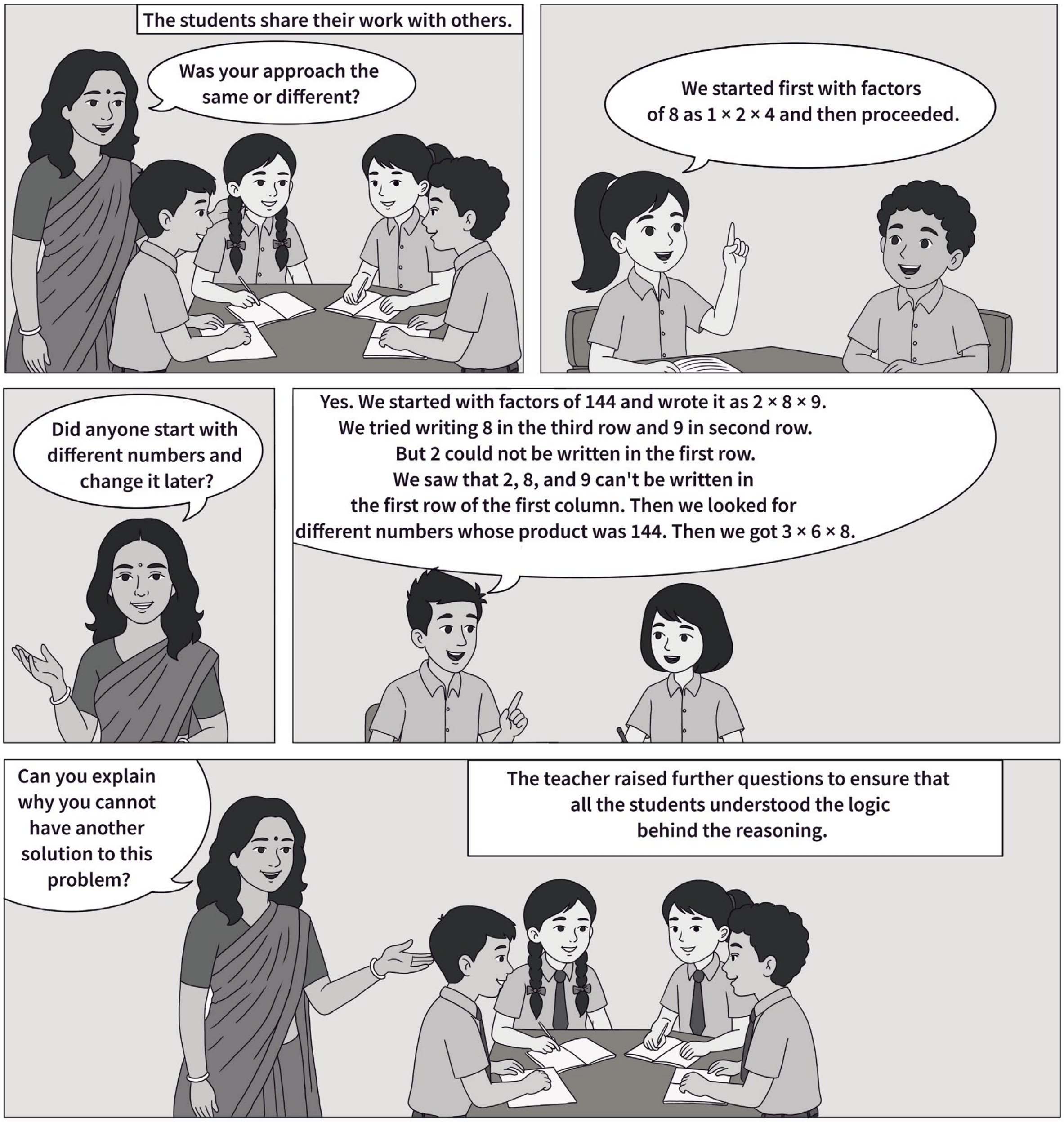



Let us just write 1 to 9 in order and see if it works.
But, if we write 1,2,3 in the top row the product will be 6. The product in the first row is 15.
So, what shall we write?
What numbers will give 15 when they are multiplied?
3 × 5 is 15. But we need three numbers.
We can use 1 as well.
OK. Let us write 1,3,5 in order.
They write in that order.
But, how will we get 8 in the foot of second column if we write 3 in this box?


Hmm. What shall we write then? 2?
No. We already said that 1,3,5 give 15 as answer.
Shall we try 1 in the second box? Now, where to write 3 and 5?
Yes. We should write 5 in the third box and 3 in the first box.
Now, we can fill Column 2. 1 × 2 × 4 = 8. Should we try 2 in the second row and 4 in the third row?
Is there a way of deciding where 2 should go and 4 should go?
Both 108 and 224 are multiples of both 2 and 4.
So, we can't decide. Let's try the third column.



Yes. Keep trying.
Let's try : 5 × 63 is 315. So, we can write 63 as 7 × 9.
Where do 7 and 9 go in the third column?
108 is a multiple of 9, but 224 is not. Therefore, we can write 9 in the second row and 7 in the third row.
3 × 48 = 144, 48 = 4 × 12, 6 × 8.
We can use 6 and 8. 6 is also a factor of 108. We will write 6 in the second row and 8 in the third row.
Right
Now, it is clear where 2 and 4 come.
Good reasoning. Will you be able to explain to the others how you reasoned out this problem?
Yes



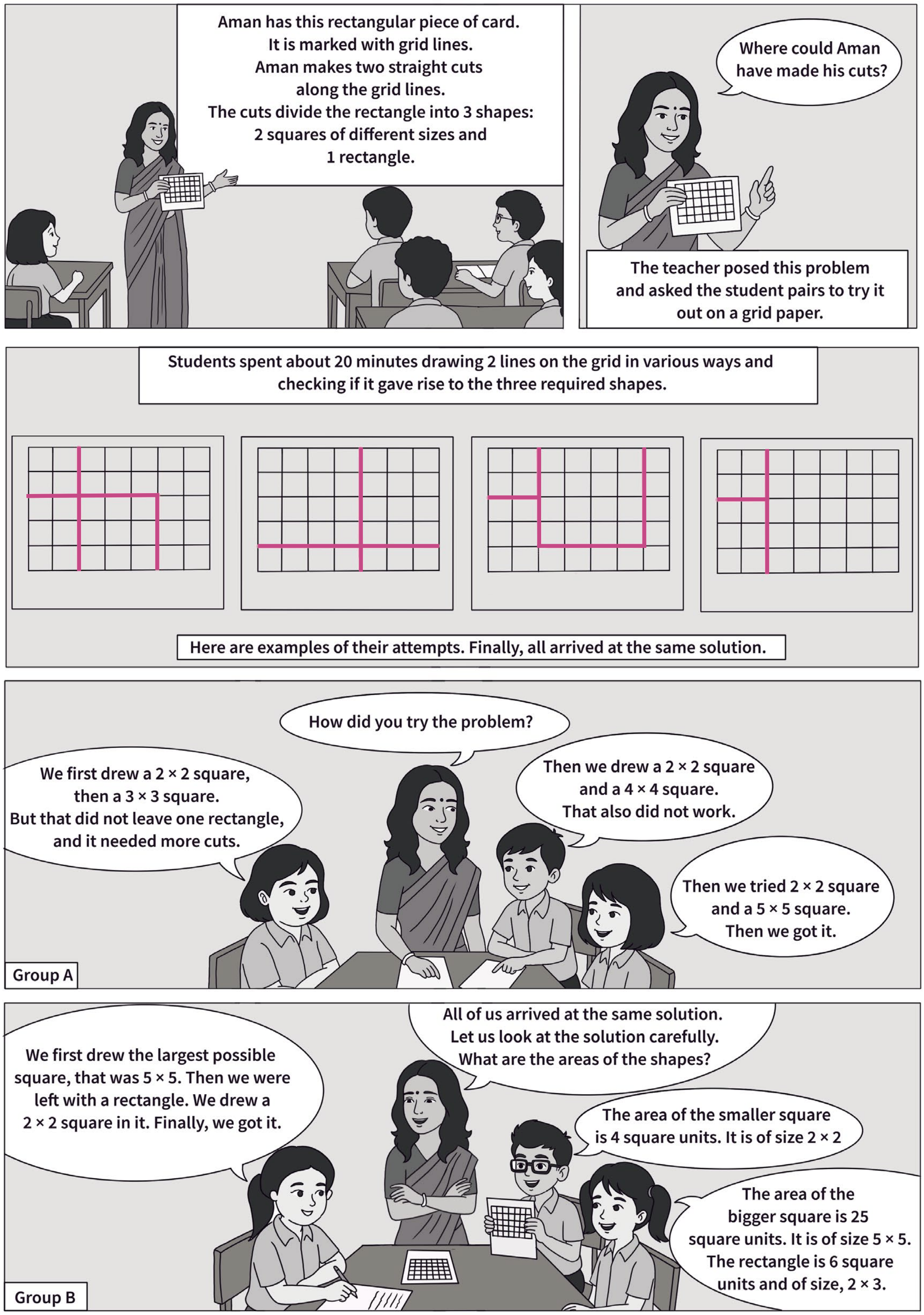
This discussion showcases most of the opportunities mentioned in Discussion 1. In addition, the students saw a connection between this problem and another.

Sometimes, relating to familiar problems can aid in understanding or solving a problem. The teacher left the first part of the discussion to the students and came into it in the middle, building a scaffold by posing questions that helped the students to proceed further.

Though there was only one solution to the problem, the emphasis was clearly on the process and how the answer could be arrived at via different routes.

DISCUSSION 3

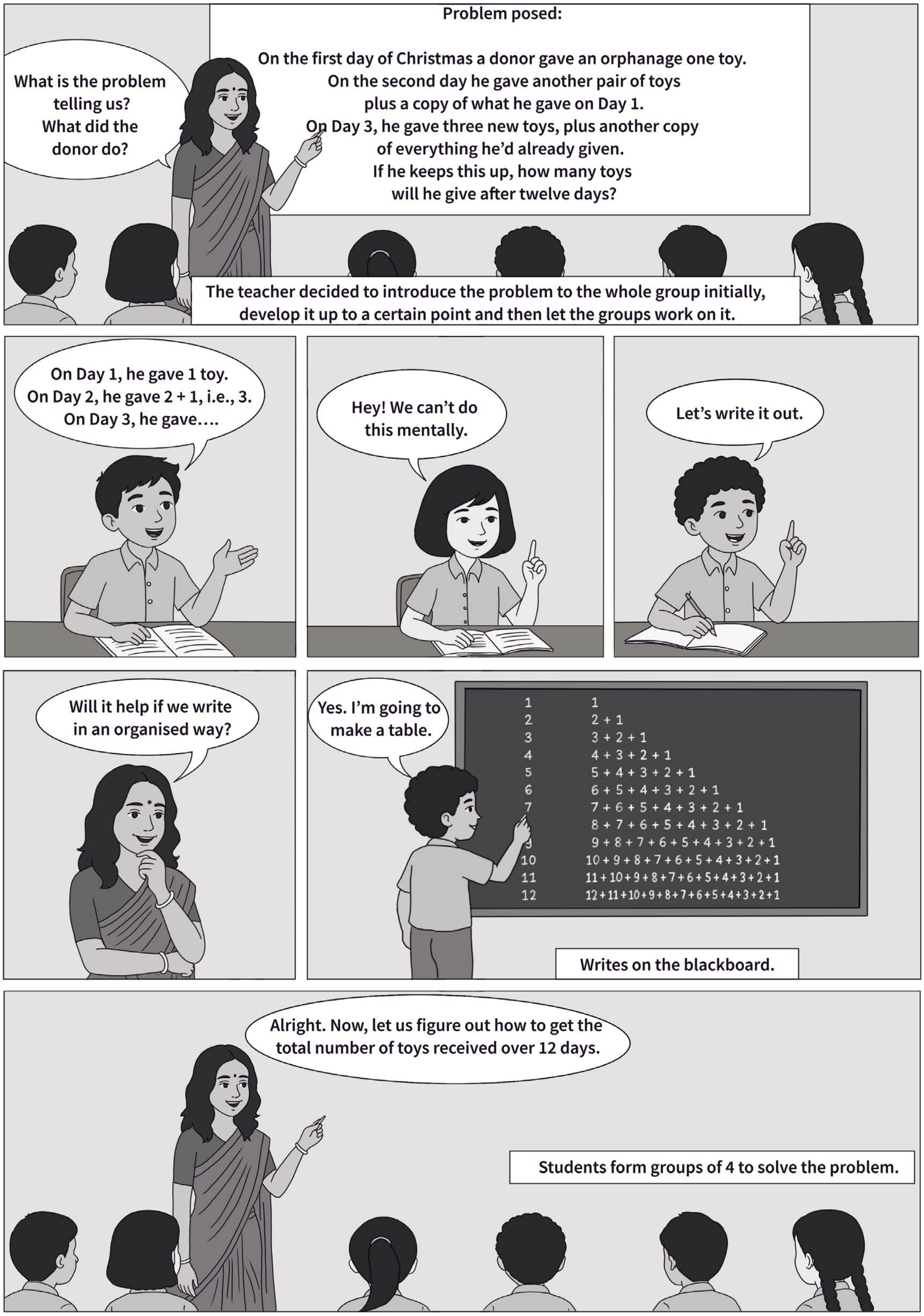
Grade Level 6, students work in groups of two or three.

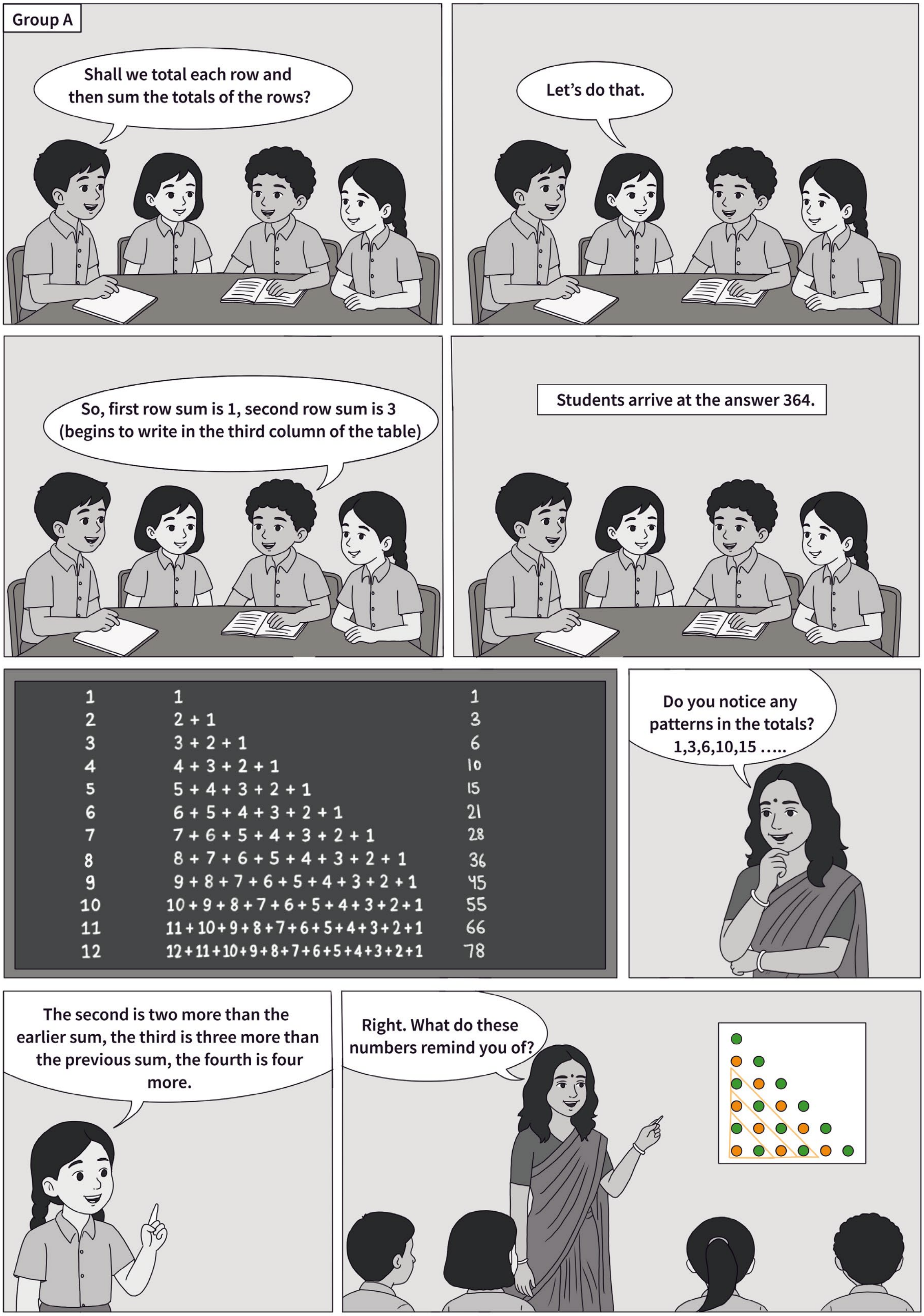


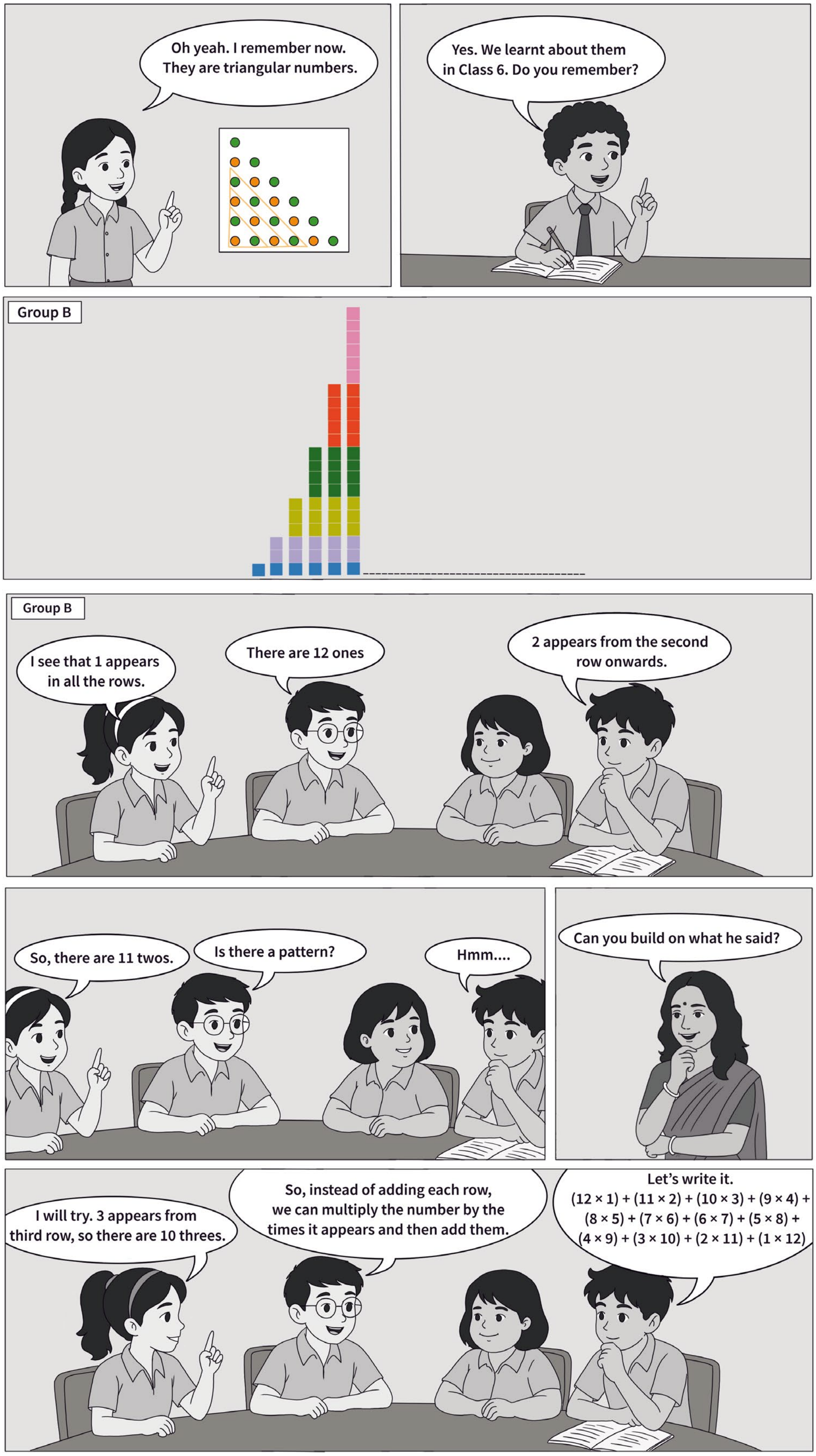
Notice that the teacher did not stop when the class arrived at one solution. What advantage do you see in the extended discussion? What are the various ways through which the teacher encouraged math talk in this discussion?

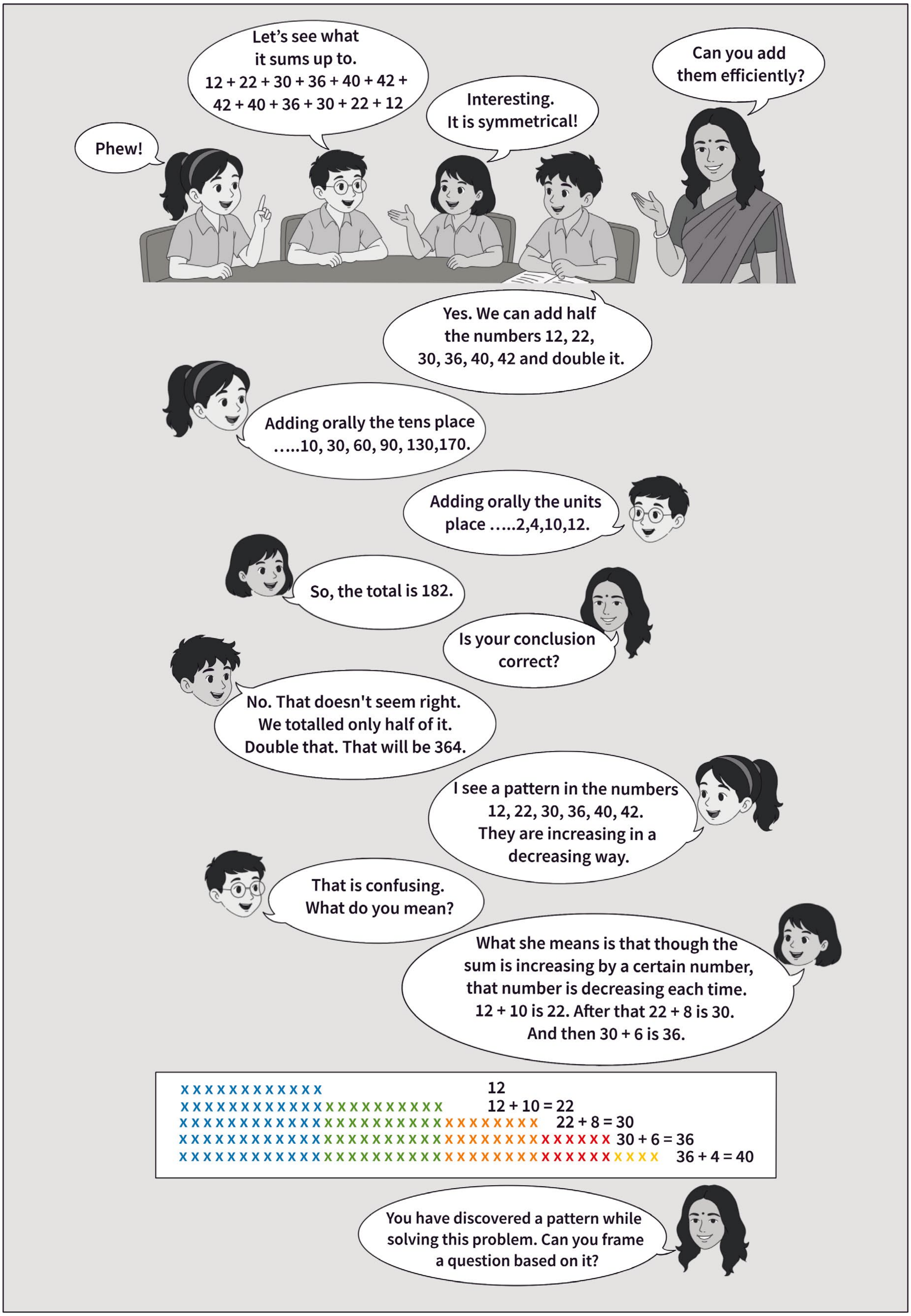
DISCUSSION 4

Grade Level 5/6, Students work in groups of 4.




Group A
Shall we total each row and then sum the totals of the rows?
Let's do that.
So, first row sum is 1, second row sum is 3 (begins to write in the third column of the table)
Students arrive at the answer 364.
1 1 1
2 2 + 1 3
3 3 + 2 + 1 6
4 4 + 3 + 2 + 1 10
5 5 + 4 + 3 + 2 + 1 15
6 6 + 5 + 4 + 3 + 2 + 1 21
7 7 + 6 + 5 + 4 + 3 + 2 + 1 28
8 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 36
9 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 45
10 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 55
11 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 66
12 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 78
Do you notice any patterns in the totals? 1,3,6,10,15 .....
The second is two more than the earlier sum, the third is three more than the previous sum, the fourth is four more.
Right. What do these numbers remind you of?

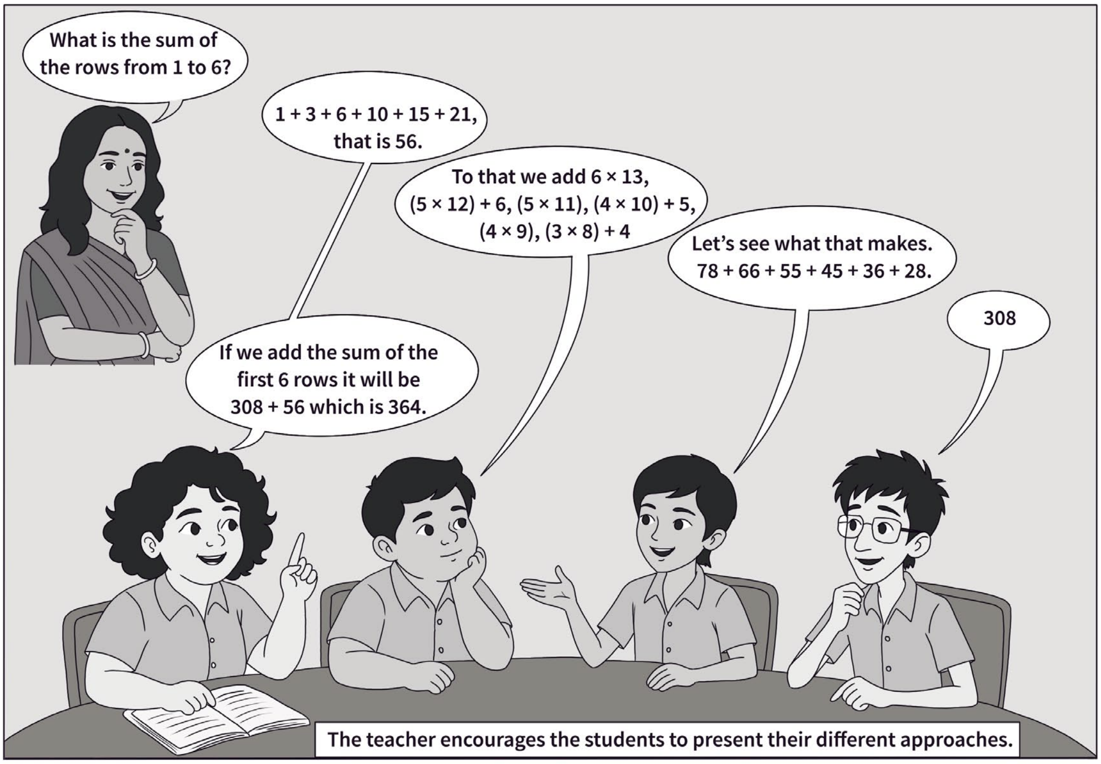

Oh yeah. I remember now. They are triangular numbers.
Yes. We learnt about them in Class 6. Do you remember?
Group B
Group B
I see that 1 appears in all the rows.
There are 12 ones
2 appears from the second row onwards.
So, there are 11 twos.
Is there a pattern?
Hmm....
Can you build on what he said?
I will try. 3 appears from third row, so there are 10 threes.
So, instead of adding each row, we can multiply the number by the times it appears and then add them.
Let's write it.
(12 × 1) + (11 × 2) + (10 × 3) + (9 × 4) +
(8 × 5) + (7 × 6) + (6 × 7) + (5 × 8) +
(4 × 9) + (3 × 10) + (2 × 11) + (1 × 12)

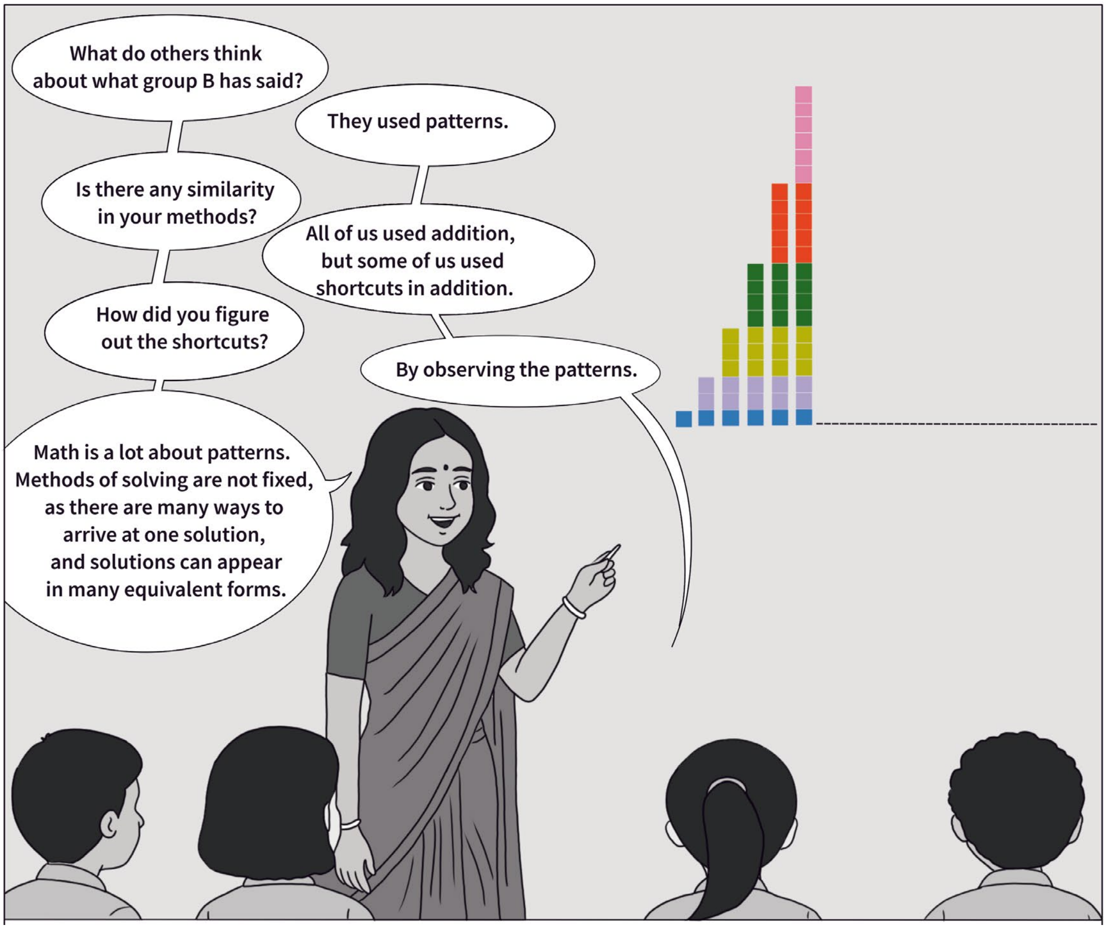


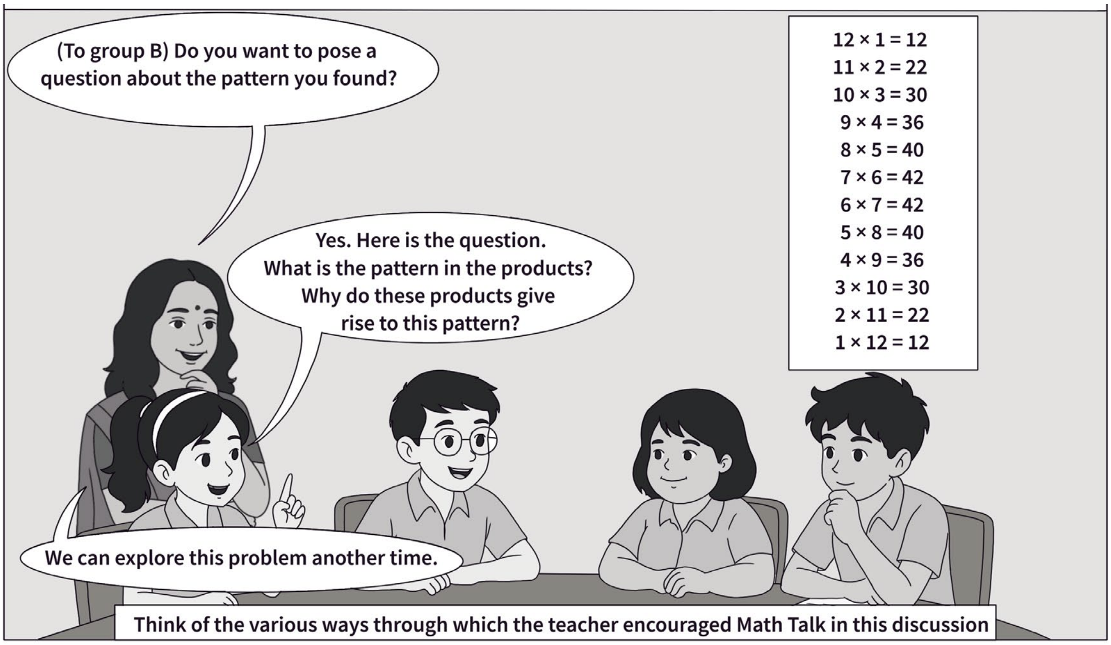
Let's see what it sums up to. 12 + 22 + 30 + 36 + 40 + 42 + 42 + 40 + 36 + 30 + 22 + 12
Phew!
Interesting. It is symmetrical!
Can you add them efficiently?
Yes. We can add half the numbers 12, 22, 30, 36, 40, 42 and double it.
Adding orally the tens place …..10, 30, 60, 90, 130,170.
Adding orally the units place …..2,4,10,12.
So, the total is 182.
Is your conclusion correct?
No. That doesn't seem right. We totalled only half of it. Double that. That will be 364.
I see a pattern in the numbers 12, 22, 30, 36, 40, 42. They are increasing in a decreasing way.
That is confusing. What do you mean?
What she means is that though the sum is increasing by a certain number, that number is decreasing each time. 12 + 10 is 22. After that 22 + 8 is 30. And then 30 + 6 is 36.
12
12 + 10 = 22
22 + 8 = 30
30 + 6 = 36
36 + 4 = 40
You have discovered a pattern while solving this problem. Can you frame a question based on it?

Group C

1	1
2	2 + 1
3	3 + 2 + 1
4	4 + 3 + 2 + 1
5	5 + 4 + 3 + 2 + 1
6	6 + 5 + 4 + 3 + 2 + 1
7	7 + 6 + 5 + 4 + 3 + 2 + 1
8	8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
9	9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
10	10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
11	11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
12	12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1


What is the sum of the rows from 1 to 6?
1 + 3 + 6 + 10 + 15 + 21, that is 56.
To that we add 6 × 13, (5 × 12) + 6, (5 × 11), (4 × 10) + 5, (4 × 9), (3 × 8) + 4
Let’s see what that makes. 78 + 66 + 55 + 45 + 36 + 28.
308
If we add the sum of the first 6 rows it will be 308 + 56 which is 364.
The teacher encourages the students to present their different approaches.


What do others think about what group B has said?
They used patterns.
Is there any similarity in your methods?
All of us used addition, but some of us used shortcuts in addition.
How did you figure out the shortcuts?
By observing the patterns.
Math is a lot about patterns. Methods of solving are not fixed, as there are many ways to arrive at one solution, and solutions can appear in many equivalent forms.



Please do share with atrightangles.editor@apu.edu.in any math talk that can be generated by these, or other examples.

We have showcased interesting questions and even more interesting discussions which allow students to discover things. A natural question would be about how practical it is for the average teacher to incorporate this, first by understanding the nuances of each question and second, by managing her time with portions to complete, deadlines to meet, and so on. In general, whenever we meet teachers, we recommend that they try such investigations and discussions at least once a month. Math talk promotes mathematical thinking and that is an important aim for the teaching of mathematics.

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PADMAPRIYA SHIRALI is part of the Community Math Centre based in Valley School (Bangalore) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. In the 1990s, she worked closely with the late Shri P K Srinivasan. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box.' She is currently part of the NCERT textbook development group. Padmapriya may be contacted at padmapriya.shirali@gmail.com