



Understanding J/ψ and ψ' production using a modified version of Non-Relativistic Quantum Chromodynamics

Sudhansu S. Biswal^{a,*}, Sushree S. Mishra^a, K. Sridhar^b

^a Department of Physics, Ravenshaw University, Cuttack 753003, India

^b School of Arts and Sciences, Azim Premji University, Sarjapura, Bangalore, 562125, India

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ABSTRACT

There is serious disagreement between the predictions of Non-Relativistic Quantum Chromodynamics (NRQCD) and the data on J/ψ polarisation which has persisted for almost a quarter of a century. We find that if we account for the effect of perturbative soft gluons on the intermediate charm-anticharm octet states in NRQCD then the polarisation problem can be resolved. In addition, this model, when used to fit the Run 1 data on J/ψ and ψ' production from the CDF experiment at Tevatron, gives good fits and yields values of (energy-independent) non-perturbative parameters. These, in turn, can be used to make *parameter-free predictions* for J/ψ and ψ' data from the CMS experiment at the Large Hadron Collider and the predictions are in excellent agreement with the CMS data.

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When Non-Relativistic Quantum Chromodynamics (NRQCD) [1] was first developed as an effective theory over twenty-five years ago, there was much hope that it would shed light on understanding quarkonium production in much the same manner that it had been already used to clarify the situation with respect to the decay of P -wave quarkonia. It was, indeed, the most opportune time to explore the consequences of this effective theory because the data from Tevatron [2] on J/ψ production had become available and showed huge disagreement with the theoretical model that preceded NRQCD – the Colour-Singlet Model (CSM) [3,4]. The success of NRQCD in bridging this massive gap between theoretical predictions and experimental data went a long way in furthering the belief of it being the correct theory of quarkonium production. But one could also discern a certain tendency among the community of NRQCD theorists of regarding it as being something of a gospel truth – the platitudinous use of the word rigorous in NRQCD papers being a marker of this tendency – but somewhere in its claims to rigour lay the seeds of its nemesis.

That was to come from the prediction of the polarisation of the produced J/ψ – from the observation that a large fraction of them produced at large transverse momentum, p_T , were from a fragmentation-like process – with a single large- p_T gluon fragmenting into an octet state (a $^3S_1^8$ state, to be precise). The appear-

ance of the colour-octet component in the gluon fragmentation function and its importance in understanding the data from the CDF experiment at the Tevatron was already explored [5] before the full understanding of the production mechanism in NRQCD was available. But, with the more complete understanding came the realisation that, in this fragmentation-like process, the $^3S_1^8 Q\bar{Q}$ state inherits the entire transverse polarisation of the (almost-real) gluon that fragments into it and, subsequently, when the $Q\bar{Q}$ state evolves non-perturbatively into a J/ψ , in the manner specified by NRQCD, the heavy-quark symmetry of NRQCD ensures that almost all of this polarisation is bequeathed to the J/ψ which, therefore, emerges transversely polarised at large- p_T [6]. This is true upto order v^2 , where v is the relative velocity between the Q and the \bar{Q} inside the bound state. This argument fails to hold at low or moderate p_T where one expects the J/ψ to be unpolarised and so the p_T dependence of the polarisation, calculated in full detail in Ref. [7], is a crucial test of the theory. The theory failed the test miserably when data from the CDF experiment [8] showed that the polarisation of the J/ψ was consistent with zero over all p_T .

It is useful to recall the basic ideas of NRQCD in somewhat more detail. NRQCD is derived from the QCD Lagrangian by neglecting all states of momenta much larger than the heavy quark mass, M_Q and to account for this exclusion by adding new interaction terms yielding the effective Lagrangian. The quarkonium state admits of a Fock-state expansion in orders of v . At leading order, the $Q\bar{Q}$ state is in a colour-singlet state but at $\mathcal{O}(v)$, it can be in a colour-octet state and connected to the physical J/ψ state through a non-perturbative gluon emission.

* Corresponding author.

E-mail addresses: sudhansu.biswal@gmail.com (S.S. Biswal), sushreesimran.mishra97@gmail.com (S.S. Mishra), sridhar.k@apu.edu.in (K. Sridhar).

The Fock space expansion of the physical J/ψ , which is a 3S_1 ($J^{PC} = 1^{--}$) state, is:

$$|J/\psi\rangle = \mathcal{O}(1) \left| Q \bar{Q} [{}^3S_1^{[1]}] \right\rangle + \mathcal{O}(v^2) \left| Q \bar{Q} [{}^3P_J^{[8]}] g \right\rangle + \mathcal{O}(v^4) \left| Q \bar{Q} [{}^1S_0^{[8]}] g \right\rangle + \mathcal{O}(v^4) \left| Q \bar{Q} [{}^3S_1^{[8]}] gg \right\rangle + \dots \quad (1)$$

In the above expansion the colour-singlet 3S_1 state contributes at $\mathcal{O}(1)$. The 3P_J contribution includes the three P states ($J = 0, 1, 2$). As the P -state production is itself down by factor of $\mathcal{O}(v^2)$ both the colour-octet P and S channels effectively contribute at the same order. The colour-octet state ${}^3P_J^{[8]}$ (${}^1S_0^{[8]}$) becomes a physical J/ψ by emitting a gluon in an E1 (M1) transition, while there is also a contribution at the same order from a ${}^3S_1^{[8]}$ state doing a double E1 transition to a J/ψ state.

The cross section for production of a quarkonium state H can be factorised as:

$$\sigma(H) = \sum_{n=\{\alpha,S,L,J\}} \frac{F_n}{M_Q^{d_n-4}} \langle \mathcal{O}_n^H ({}^{2S+1}L_J) \rangle, \quad (2)$$

where F_n 's are the short-distance coefficients and \mathcal{O}_n are operators of naive dimension d_n , describing the long-distance effects. These non-perturbative matrix elements are guaranteed to be energy-independent due to the NRQCD factorization formula, so that they may be extracted at a given energy and used to predict quarkonium cross-sections at other energies.

Writing this down explicitly in terms of the various octets and singlet intermediate states gives:

$$\sigma_{J/\psi} = \hat{F}_{3S_1^{[1]}} \times \langle \mathcal{O}({}^3S_1^{[1]}) \rangle + \hat{F}_{3S_1^{[8]}} \times \langle \mathcal{O}({}^3S_1^{[8]}) \rangle + \hat{F}_{1S_0^{[8]}} \times \langle \mathcal{O}({}^1S_0^{[8]}) \rangle + \frac{1}{M^2} \left[\hat{F}_{3P_J^{[8]}} \times \langle \mathcal{O}({}^3P_J^{[8]}) \rangle \right]. \quad (3)$$

In Refs. [9,10] the complete set of short-distance coefficients in NRQCD needed to study J/ψ and χ production was calculated and compared with the data from Tevatron.¹ These NRQCD calculations gave a good description of the shapes of the p_T distributions of the charmonium resonances at the Tevatron but the normalization of these distributions was not predicted in NRQCD i.e. the non-perturbative matrix elements which determined the normalization had to be obtained by a fit to the data. Independent tests of the effective theory approach were, therefore, necessary to determine the validity of the approach and, indeed, other than the polarisation of the J/ψ various proposals were made [13] to test NRQCD. But of all these, polarisation was the litmus test of the theory and the serious disagreement of NRQCD predictions with data on J/ψ polarisation calls for a critical evaluation of the theory.

It could be that the reason that the polarisation predictions are going awry is that the charm quark is too light to be treated in NRQCD. However, if that is the reason one cannot understand why the cross-sections for the charmonium resonances work out right in NRQCD. The other line of attack has been to first note that the colour-singlet model predicts zero polarisation and to then attempt to jack up the colour-singlet contribution by invoking higher-order effects in the singlet channel [14,15].² From the point of view of the effective theory, it makes little sense to leave out one set of operators (colour-octet) and work with only the other set (colour-singlet) without any argument for the smallness of the octet operators and their effects in charmonium production. The next-to-leading order corrections for the NRQCD matrix elements

have been computed [18] but then that does little to suggest a way out of the polarisation problem.

In the colour-singlet model, the $c\bar{c}$ pair is produced in the colour-singlet state with the appropriate angular momentum and spin assignments and this forms the physical quarkonium state through a non-perturbative transition. The description of the bound-state in NRQCD and, indeed, the computational procedure to obtain the short-distance coefficients F_n is the same as in the colour-singlet model: the $c\bar{c}$ pair could be in a colour-singlet or octet state and, if it is in an octet state it emits one or more *non-perturbative* gluons to make a transition to the physical quarkonium state. The fact that a fixed number (one or two, in practice) of gluons mediate this transition it is possible to figure out what the quantum numbers of the octet $c\bar{c}$ would be given the quantum numbers of the quarkonium state. In our proposed modification of the picture, this is the point that we wish to deviate from the usual narrative. The colour-octet $c\bar{c}$ state can radiate several soft *perturbative* gluons – each emission taking away little energy but carrying away units of angular momentum. In the multiple emissions that the colour-octet state can make before it makes the final NRQCD transition to a quarkonium state, the angular momentum and spin assignments of the $c\bar{c}$ state changes constantly. Perturbative soft gluon emission from colour-octet states to address the polarisation problem has also been used in Ref. [19] but it differs considerably from the approach we present here. We also present a modified cross-section formula with which we are able to fit the Tevatron cross-sections and present excellent predictions for the LHC cross-sections measurements.

Let us consider J/ψ production, to be specific. When an octet state transforms into a J/ψ that process happens in NRQCD via a non-perturbative gluon and is subject to the counting rules and symmetries of NRQCD. The gluons that we are invoking are perturbative soft gluons that affect the intermediate perturbative state between the short-distance process and the final NRQCD transition. If one uses the NRQCD rules to pin down what octet states can transform into a J/ψ and also neglect the short-distance production of higher angular momentum states then the only octet states at the short-distance level that we need to consider are ${}^3S_1^{[8]}$, ${}^3P_{0,1,2}^{[8]}$, ${}^1S_0^{[8]}$ and ${}^1P_1^{[8]}$. If we label these states as S_i , $i = 1, \dots, 6$ then the soft gluon emission process can be thought of as a stochastic process that mediates transitions between these several states S_i .

The resummation of leading logarithms resulting from the soft-gluon emission will give rise to the familiar Sudakov form factor but the angular momentum of the radiating object is left unchanged at this level. However, higher-order power-suppressed terms change this picture – the amplitude when expanded in powers of the threshold parameter yields the angular-momentum operator and such terms can change the polarisation of the radiating coloured particle [20–22].

In particular, if at large- p_T we were to produce a ${}^3S_1^{[8]} \equiv S_1$ state in a short-distance fragmentation-like process, this state will now emit several soft gluons which will obliterate the transverse polarisation that the state was produced with because the intermediate states S_i produced in the process of soft-gluon emission will randomly oscillate between transverse and longitudinal polarisation, yielding a net polarisation which is zero. The perturbative soft gluons, thus, can give the right prediction for the polarisation of the J/ψ . We do not attempt to calculate the dynamics of the soft-gluon emission explicitly – but, indeed, in the approach we follow we do not need to. It is sufficient to know that with each emission the angular momentum of the coloured state can change. What we compute explicitly are the cross-sections and for that we do not need a detailed understanding of the soft dynamics, as we show below.

¹ See also Ref. [11]. For a detailed review of quarkonium production see Ref. [12].

² For reviews of the status of these calculations and their experimental consequences, see Refs. [16,17].

The price that has to be paid for bring in the perturbative soft gluons, however, is that we seem to lose the ability to pin down the quantum numbers of the intermediate $c\bar{c}$ state. In the usual picture, this was well under control: the state produced in the perturbative process was the one that transitioned into the J/ψ finally. With the soft gluons brought in, this is no longer true. But we will see that, in spite of this apparent problem, we have enough information to be able to compute the cross-sections. Because of the stochastic mixing between the states S_i , the cross-section formula in Eqn. (3) no longer holds. If we assume our process is Markovian and the states S_i freely mix with all the transition probabilities being equal then we can write down a cross-section formula as follows:

$$\begin{aligned} \sigma_{J/\psi} = & \left[\hat{F}_{3S_1^{[1]}} \times \langle \mathcal{O}^3 S_1^{[1]} \rangle \right] \\ & + \left[\hat{F}_{3S_1^{[8]}} + \hat{F}_{1P_1^{[8]}} + \hat{F}_{1S_0^{[8]}} + \hat{F}_{3P_J^{[8]}} \right] \times \left(\frac{\langle \mathcal{O}^3 S_1^{[1]} \rangle}{8} \right) \\ & + \left[\hat{F}_{3S_1^{[8]}} + \hat{F}_{1P_1^{[8]}} + \hat{F}_{1S_0^{[8]}} + \hat{F}_{3P_J^{[8]}} \right] \times \langle \mathcal{O} \rangle, \end{aligned} \quad (4)$$

where

$$\langle \mathcal{O} \rangle = \times \left[\langle \mathcal{O}^3 S_1^{[8]} \rangle + \langle \mathcal{O}^1 S_0^{[8]} \rangle + \frac{\langle \mathcal{O}^3 P_J^{[8]} \rangle}{M^2} \right] \quad (5)$$

In contrast to the usual case, where we needed to fix three non-perturbative parameters to get the J/ψ cross-section, in our case it is the sum of these parameters: so we have a single parameter to fit.

To test the model that we have proposed, we start by using this cross-section formula to make a fit to the data on J/ψ and ψ' production from the CDF experiment at Tevatron [2]. The p_T distribution is given by the standard formula:

$$\begin{aligned} \frac{d\sigma}{dp_T} (p\bar{p} \rightarrow c\bar{c} [^{2S+1}L_J^{[1,8]}] X) = \\ \sum \int dy \int dx_1 x_1 G_{a/p}(x_1) x_2 G_{b/p}(x_2) \frac{4p_T}{2x_1 - \bar{x}_T e^y} \\ \frac{d\hat{\sigma}}{dt} (ab \rightarrow c\bar{c} [^{2S+1}L_J^{[1,8]}] d), \end{aligned} \quad (6)$$

where the summation is over the partons (a and b), the final state $Q\bar{Q}$ is in the $^1S_0^{[1]}$, $^1P_1^{[8]}$, $^3S_1^{[8]}$ states and $G_{a/p}$, $G_{b/p}$ are the distributions of partons a and b in the protons and x_1 , x_2 are the respective momentum they carry. In the above formula, $\bar{x}_T = \sqrt{x_T^2 + 4\tau} \equiv 2M_T/\sqrt{s}$ with $x_T = 2p_T/\sqrt{s}$ and $\tau = M^2/s$. \sqrt{s} is the centre-of-mass energy, M is the mass of the resonance and y is the rapidity at which the resonance is produced. The matrix elements for the subprocesses corresponding to $F_1[^1S_0]$, $F_8[^3P_J]$ and $F_8[^3S_1]$ are listed in Refs. [10,23]. The remaining coefficient $F_8[^1P_1]$ has been calculated in [24].

We assume, and it is reasonable to do so, that the p_T distributions of the final J/ψ is not significantly different compared to that of the octet state produced at the short-distance level. The multiple gluon emission would at best change the normalisation of the cross-section somewhat. We are fitting the normalisation, in any case, by comparing our theoretical predictions to the data from Tevatron and so the effect of the soft-gluons is accounted for in the fit. The fits to the 1.8 TeV Tevatron data on J/ψ and ψ' production are shown in Fig. 1. Good fits to both sets of data are obtained.

The true test of our model is to see what predictions are obtained for J/ψ and ψ' production at the Large Hadron collider

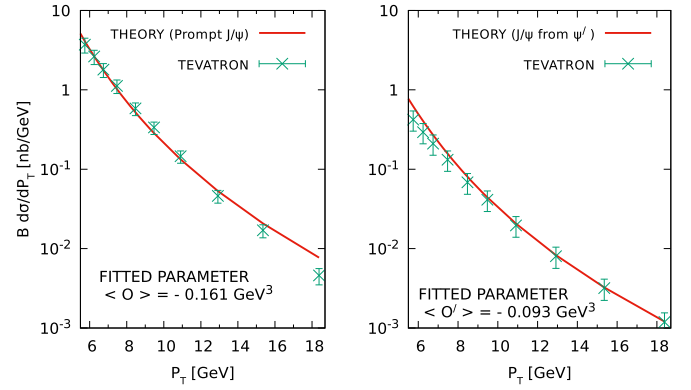


Fig. 1. Theoretical prediction of differential cross sections fitted to the data on J/ψ and ψ' production from the CDF experiment at Tevatron.

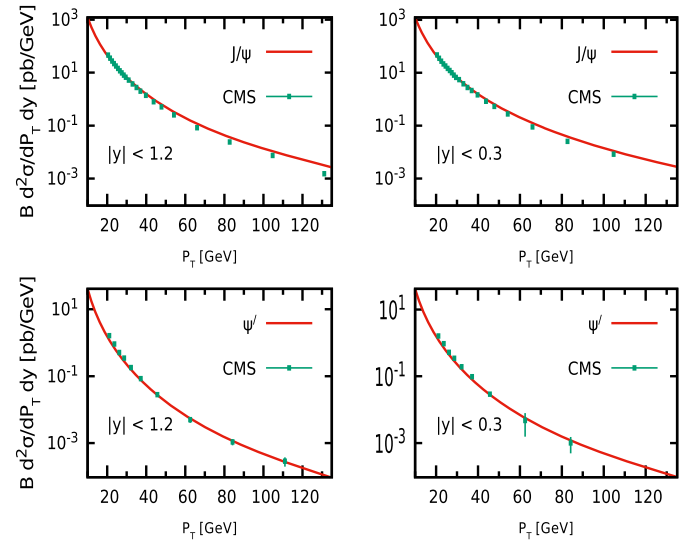


Fig. 2. Predicted differential distributions for J/ψ and ψ' production at the LHC running at 13 TeV compared with data from the CMS experiment.

runs of 13 TeV. We use the fitted non-perturbative parameters (one each for J/ψ and ψ') to predict the distributions (in p_T and in bins of y) and compare them with data from the CMS experiment [25]. As can be seen from, Fig. 2, the predictions compare very well with the CMS data.

In conclusion, provoked by the long-standing disagreement of the predictions of Non-Relativistic QCD (NRQCD) with the data on J/ψ polarisation, we have critically examined the theory and found it justified to study the effect, neglected hitherto, of soft-gluon with the colour-octet $c\bar{c}$ pair that eventually forms a charmonium state like J/ψ . In complete contrast to the usual picture, our model predicts that the produced J/ψ is unpolarised which is in agreement with the polarisation data from Tevatron. We have then fitted our model predictions to Tevatron data on J/ψ and ψ' production and used the fitted parameters to *predict* the distributions at the LHC energy, and find excellent agreement with data from the CMS experiment.

The results are very encouraging and more studies of this model to understand the production of other charmonium resonances and in other experimental situations are being planned.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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