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Letter

Predictions for h_c and h_b production at the LHC

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ABSTRACT

The production cross sections of h_c and h_b, the $^1P_1$ quarkonia can be predicted in Non-Relativistic Quantum Chromodynamics (NRQCD) using heavy quark symmetry. Our study includes predictions for both the integrated cross-section and the transverse momentum (p_T) distribution of h_c (h_b) production at the Large Hadron Collider (LHC), using the decay process $h_c(h_b) \to \eta_c(\eta_b) + \gamma$, $\eta_c(\eta_b) \to p\bar{p}$. We demonstrate substantial discrepancies in integrated cross-section and the p_T distribution of $^1P_1$ quarkonia production at the LHC using Colour-Singlet Model (CSM), NRQCD and modified NRQCD. Measuring these resonances at the LHC could discriminate between these models, thereby offering further insights into the dynamics of quarkonium production. In addition, we compare the recent LHCb data for the integrated cross-section of h_c production at $\sqrt{s} = 13$ TeV in the kinematic range $5.0 < p_T < 20.0$ GeV and $2.0 < y < 4.0$ with the theoretical predictions using NRQCD and modified NRQCD. Modified NRQCD gives an agreement with the recent LHCb experimental data.

The discovery of the first quarkonium state, J/ψ in 1974 has significantly expanded our understanding of quarkonium properties and the theoretical concepts and methods based on Quantum Chromodynamics (QCD). Potential models [1] explain all charmonium states below the $D\overline{D}$ threshold, as observed experimentally. The final charmonium state among these charmonium states below the $D\overline{D}$ threshold has been experimentally verified is the P-wave spin-singlet state h_c ($^1P_1$). In 1992, the E760 Collaboration at Fermilab [2] first observed the h_c ($^1P_1$) state through $p\bar{p}$ annihilation. In recent years, some experimental measurements of P-wave quarkonia, including the h_c and h_b (1^{+-}) ($^1P_1$ charmonium and bottomonium states) have been conducted. The corresponding branching ratios [3], the masses of these quarkonia [4–8] and the cross sections for h_c (h_b) production through e^+e^- annihilation at the CLEO-c [7] and B-factories [8] have been measured experimentally. In contrast, only leading-order (LO) results have been provided for h_c and h_b production. Calculations of h_c hadroproduction at the Tevatron [9] and the LHC [10–12] predicted a substantial yield. The photoproduction of h_c was examined in [13] using a color-octet (CO) long-distance matrix element (LDME) derived from the decay $B \to \chi_{cJ} + X$. The results suggested a substantial cross section at DESY HERA. h_c (h_b) production via e^+e^- annihilation [14,15] and by the B factory [16–18] has been studied.

The lack of research on h_c and h_b suggests that the significance of these mesons has been neglected. First of all, the hadroproduction rate of h_c serves as an excellent test of Non-Relativistic Quantum Chromodynamics (NRQCD) [19]. This is due to the fact that NRQCD predictions are substantially higher than those based on the Colour-Singlet Model (CSM) [20,21], providing an adequate comparison with experimental observations. Additionally, the cross sections for h_c depend on only one nonperturbative parameter, unlike J/ψ, where the precise determination of three color-octet long distance matrix elements introduces ambiguity. According to the NRQCD scaling rule, the CO LDME for h_c must have the same magnitude as χ_{c1}. This rule provides an opportunity to test the corresponding velocity scaling rule as well as explore h_c within the NRQCD framework in spite of the lack of experimental data.

NRQCD has been more successful in explaining the systematics of quarkonium production at the Fermilab Tevatron [22,23], compared to the then existing CSM, which was used to analyze the production of quarkonia, where the $Q\bar{Q}$ state produced in the short-distance process was assumed to be a color-singlet. NRQCD predicts transverse polarization at high p_T, but experiments fail to see any evidence for the polarization in J/ψ [24] or Υ [25–27] measurements. Therefore, independent tests of NRQCD [9,10,23,28–31] are consequently important and the prediction of polarization of the produced quarkonium state is an important test.

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Another important test of NRQCD comes from the study of quarkonia using heavy-quark symmetry. In particular, the non-perturbative parameters required for η_c, η_b, h_c and h_b production can be obtained, using heavy quark symmetry, from the parameters of J/ψ, Υ, χ_c and χ_b production respectively. This approach has been used to predict the η_c, η_b, h_c and h_b production cross-sections at the Tevatron [9,29] and at the LHC [10,30,32]. The NRQCD predictions for η_c production have shown a significant disagreement when compared with experimental data from LHCb [33,34], resulting a notable failure. As there was a major conflict shown between NRQCD predictions and observed experimental data, NRQCD required some modification to address quarkonium production fruitfully.

The cross-section for the production of a quarkonium state H is given as:

$$\sigma(H) = \sum_{n=\{\alpha,S,L,J\}} \frac{F_n}{M^{d_n-4}} \langle \mathcal{O}_n^H(^{2S+1}L_J)\rangle, \tag{1}$$

where F_n's are the short-distance coefficients, which correspond to the production of $Q\bar{Q}$ in the angular momentum and color state denoted by n, determined through perturbative QCD calculations. The non-perturbative matrix elements, $\mathcal{O}_n$ are operators of naive dimension d_n, which describe the long-distance physics are not calculable and have to be obtained by fitting to available data.

In a recently proposed modification of NRQCD [32,35–37], the color-octet $Q\bar{Q}$ state can radiate several soft *perturbative* gluons – each emission taking away little energy but carrying away units of angular momentum. In the multiple emissions that the color-octet state can make before it makes the final NRQCD transition to a quarkonium state, the angular momentum and spin assignments of the $Q\bar{Q}$ state changes constantly. The LHC data on η_c production strongly disagreed with the predictions of NRQCD but was in very good agreement with those of modified NRQCD. Moreover the η_b production has been predicted in both modified NRQCD and NRQCD. In contrast to the case of η_c, however, the NRQCD predictions for η_b and the color-singlet prediction are very similar except maybe at very large p_T and the modified NRQCD prediction is very different from both these predictions. Motivated by these observations, we would like to study h_c and h_b production in both NRQCD and modified NRQCD. Measurement of this production at the LHC will provide a very good test of both NRQCD and modified NRQCD.

The fock-state expansion of quarkonium states is characterized by v, the relative velocity between quark and anti-quark pair. At leading order, the $Q\bar{Q}$ state is in a color-singlet state but at $\mathcal{O}(v)$, it can be in a color-octet state and connected to the physical h_c state through the emission of non-perturbative gluon. In NRQCD, the Fock space expansion of the physical h_c, which is a $^1P_1$ ($J^{PC} = 1^{+-}$) state, can be written as:

$$|h_c\rangle = \mathcal{O}(1)\left|c\bar{c}[^1P_1^{[1]}]\right\rangle + \mathcal{O}(v^2)\left|c\bar{c}[^1S_0^{[8]}]\,g\right\rangle + \cdots \,. \tag{2}$$

In the above equation, the color singlet $^1P_1^{[1]}$ state is associated with the order of relative velocity $\mathcal{O}(1)$, while the contribution of the color-octet $^1S_0^{[8]}$ state occurs at the order $\mathcal{O}(v^2)$. The color-octet state $^1S_0^{[8]}$ transforms into the physical state h_c ($^1P_1^{[1]}$) through the emission of a gluon in an E1 transition.

The NRQCD cross-section formula for h_c production can be written down explicitly in terms of octet and singlet intermediate states. The cross-section for h_c production can be represented as:

$$\sigma_{h_c} = \hat{F}_{^1P_1^{[1]}} \times \langle \mathcal{O}^{h_c}(^1P_1^{[1]})\rangle + \hat{F}_{^1S_0^{[8]}} \times \langle \mathcal{O}^{h_c}(^1S_0^{[8]})\rangle + \cdots . \tag{3}$$

The coefficients F_n's represent the cross sections for producing a $c\bar{c}$ pair in the angular momentum and color states, denoted by n. The above NRQCD formula gets modified to the following in the modified NRQCD with perturbative soft gluon emission:

$$\begin{aligned}\sigma_{h_c} &= \hat{F}_{^1P_1^{[1]}} \times \langle \mathcal{O}^{h_c}(^1P_1^{[1]})\rangle \\ &+ \left[\hat{F}_{^3S_1^{[8]}} + \hat{F}_{^1P_1^{[8]}} + \hat{F}_{^1S_0^{[8]}} + \hat{F}_{^3P_J^{[8]}}\right] \times \left(\frac{\langle \mathcal{O}^{h_c}(^1P_1^{[1]})\rangle}{8}\right) \\ &+ \left[\hat{F}_{^3S_1^{[8]}} + \hat{F}_{^1P_1^{[8]}} + \hat{F}_{^1S_0^{[8]}} + \hat{F}_{^3P_J^{[8]}}\right] \times \langle \mathcal{O}^{h_c}\rangle,\end{aligned} \tag{4}$$

where

$$\langle \mathcal{O}^{h_c}\rangle = \times\left[M^2\langle\mathcal{O}(^3S_1^{[8]})\rangle + M^2\langle\mathcal{O}(^1S_0^{[8]})\rangle + \langle\mathcal{O}(^3P_J^{[8]})\rangle\right]. \tag{5}$$

In contrast to the NRQCD, we needed to fix three non-perturbative parameters to get the h_c cross-section, but in modified NRQCD, it is the sum of these parameters: so we have a single parameter to fit. Similar formulation is being made for h_b production.

Using heavy-quark symmetry relations, the non-perturbative parameters for h_c and h_b can be determined from χ_c and χ_b respectively. We have used the following heavy-quark symmetry relations [38]:

$$\langle \mathcal{O}^{h_c}(^1P_1^{[1]})\rangle = 3 \times \langle \mathcal{O}^{\chi_{c0}}(^3P_J^{[1]})\rangle = 0.321\ GeV^5\ [38], \tag{6}$$

$$\langle \mathcal{O}^{h_c}(^1S_0^{[8]})\rangle = 3 \times \langle \mathcal{O}^{\chi_{c0}}(^3S_1^{[8]})\rangle = 0.0066\ GeV^3\ [38], \tag{7}$$

$$\langle \mathcal{O}^{h_b}(^1P_1^{[1]})\rangle = 3 \times \langle \mathcal{O}^{\chi_{b0}}(^3P_J^{[1]})\rangle = 7.2\ GeV^5\ [23], \tag{8}$$

$$\langle \mathcal{O}^{h_b}(^1S_0^{[8]})\rangle = 3 \times \langle \mathcal{O}^{\chi_{b0}}(^3S_1^{[8]})\rangle = 0.045\ GeV^3\ [23], \tag{9}$$

$$\left\langle \mathcal{O}^{h_c}\right\rangle = \langle \mathcal{O}^{\chi_c}\rangle, \tag{10}$$

$$\left\langle \mathcal{O}^{h_b}\right\rangle = \langle \mathcal{O}^{\chi_b}\rangle, \tag{11}$$

where $\langle \mathcal{O}^{\chi_c}\rangle$ and $\langle \mathcal{O}^{\chi_b}\rangle$ are the fitted parameters for χ_c and χ_b respectively. We have taken $\langle \mathcal{O}^{\chi_c}\rangle = -0.0107$ GeV^5 [36] and $\langle \mathcal{O}^{\chi_b}\rangle = -0.3$ GeV^5 [32], which were obtained earlier using modified NRQCD. The matrix elements for the subprocesses are listed in Refs. [39,40].

Figs. 1 and 2 represent the h_c production differential cross-sections as a function of p_T and integrated cross-sections for different p_T-cuts in both modified NRQCD and NRQCD. Similarly, Figs. 3 and 4 are for h_b production. Here we study h_c in its decay into η_c and γ with a branching fraction of 60 % and η_c decays into $p\bar{p}$ state with a branching fraction of 1.33×10^{-3} [41]. For h_b production, we consider its decay into η_b and γ taking the branching fraction of 52 % and a 1.33×10^{-3} $p\bar{p}$ decay branching fraction for η_b.[1] Singlet prediction is also shown in all the figures. As the modified NRQCD prediction is very different from both NRQCD and singlet prediction, a cross-section measurement of the $^1P_1$ production at the LHC experiments can provide a crucial test of this interesting set of predictions. To get a sense of the feasibility of measuring the h_c and h_b production at the LHC, we have also calculated the p_T-integrated cross-sections in two different rapidity ranges. These numbers are presented in Table 1 for an integrated luminosity of 2 fb^{-1} and suggest that one should expect a sizeable number of h_c and h_b events at the LHC experiments.

The LHCb experiment has recently published some new results on integrated cross-section for h_c production [42] at $\sqrt{s}$ = 13 TeV in the kinematic range $5.0 < p_T < 20.0$ GeV and $2.0 < y < 4.0$. Therefore, we have compared the LHCb results for $h_c(1P)$ prompt production cross-section with theoretical prediction using both NRQCD and modified NRQCD. We have obtained an integrated cross-section for h_c production using NRQCD is 0.007 nb, while the modified NRQCD approach gives the value of 0.371 nb. Moreover, we have observed that the predictions of h_c production for modified NRQCD are in agreement with the measured LHCb data.

In conclusion, we have studied the $^1P_1$ quarkonia production in both modified NRQCD and NRQCD, using the heavy-quark symmetry of NRQCD. As a model discriminating observable, we suggest the measurements of the integrated cross-section and the p_T distribution of the $^1P_1$ quarkonia production at the LHC. We show that there are huge

[1] we have taken the branching ratio of $\eta_c \to p\bar{p}$ as the branching ratio of $\eta_b \to p\bar{p}$ is not available.

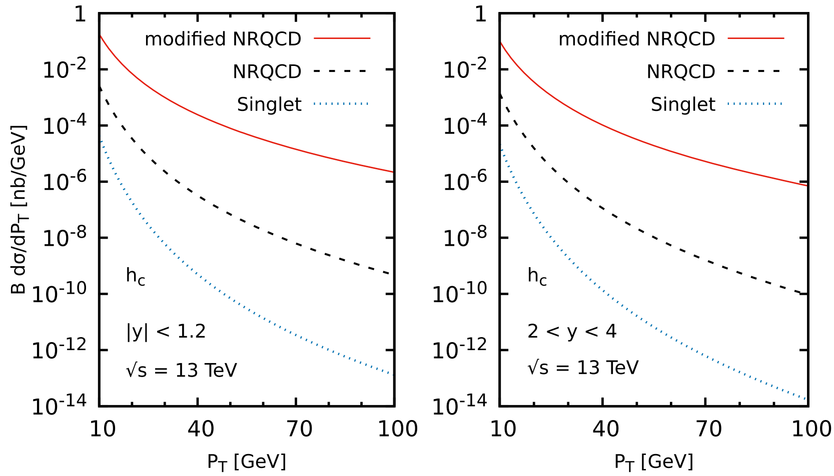

Fig. 1. Differential cross-section for h_c production at the 13 TeV LHC.

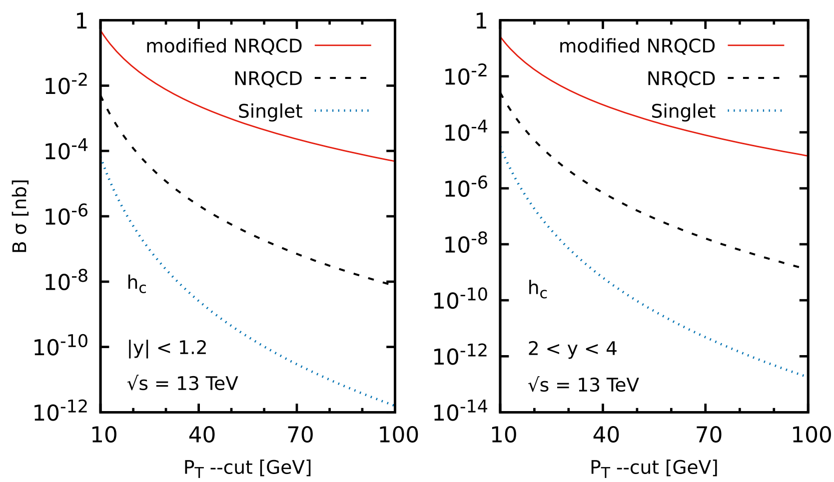

Fig. 2. Integrated cross-section for h_c production at the 13 TeV LHC.

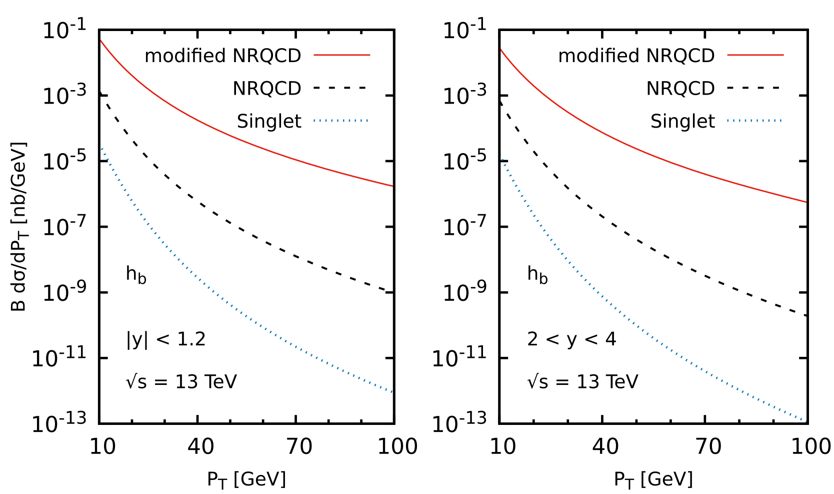

Fig. 3. Differential cross-section for h_b production at the 13 TeV LHC.

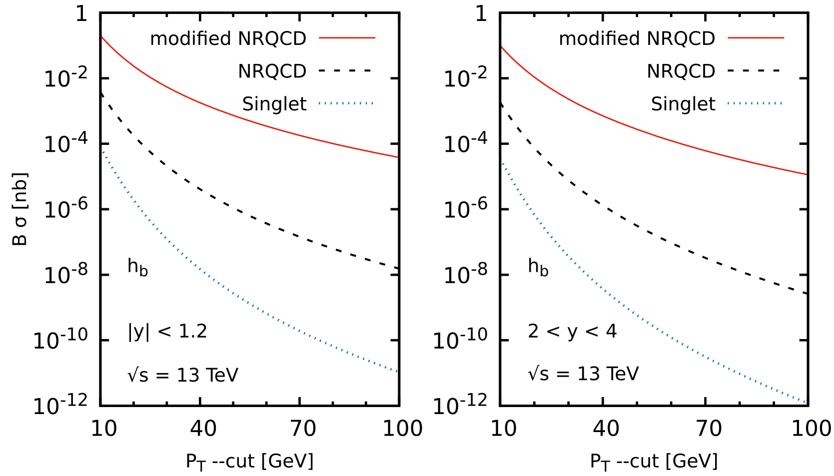

Fig. 4. Integrated cross-section for h_b production at the 13 TeV LHC.

Table 1
Number of $^1P_1$ production events expected at the LHC running at $\sqrt{s}$ = 13 TeV.

Rapidity range	p_T range (GeV)	~ Expected number of events			
		h_c production		h_b production	
		NRQCD prediction	Modified NRQCD prediction	NRQCD prediction	Modified NRQCD prediction
2 < y < 4	5 < p_T < 15	1.4×10^5	6.6×10^6	2.9×10^4	9.1×10^5
-1.2 < y < 1.2	10 < p_T < 100	1.0×10^4	9.8×10^5	7.5×10^3	4.0×10^5

differences in the integrated cross-section and the p_T distribution of $^1P_1$ quarkonia in the two models. These resonances should be measured in order to distinguish between modified NRQCD and NRQCD and to provide additional insight into the dynamics of quarkonium production. Furthermore, while comparing with the recent data from the LHCb experiment for the integrated cross-section of h_c production with the theoretical predictions using NRQCD and modified NRQCD, we find that modified NRQCD gives an agreement with data from the LHCb experiment.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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